# Theoretical Analysis of Resonant Frequency for Anisotropic Artificial Circular Dielectric Resonator Encapsulated in Waveguide

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Abstract: The analysis of resonant frequency for anisotropic artificial circular dielectric resonator (CDR) encapsulated in waveguide is investigated theoretically. The anisotropic permittivity of CDR is established by taking a relative permittivity value in one direction higher than the values of other directions in cylindrical coordinate system. By deriving Maxwell time-dependent curl equations for the CDR inside of a short-ended circular waveguide and then applying proper boundary conditions for the waveguide walls, mathematical formulation to calculate resonant frequencies for transverse electric (TE) and transverse magnetic (TM) wave modes as the function of material thickness and the anisotropic permittivity value are determined. For a comparison, the analysis is also performed for conventional CDR loaded in the same waveguide. In this case, the conventional CDR uses a natural dielectric material with isotropic permittivity. From the results, it shows that the anisotropic artificial CDR has resonant frequencies lower than the conventional CDR for the first of 3 successive TE and TM wave modes. The significant impact in lowering resonant frequencies for the TE and TM wave modes are shown by the anisotropic permittivity in  $\rho$ - and z-directions, respectively. The anisotropic permittivities are able to reduce the resonant frequencies of conventional CDR up to 13.77%, 4.19%, and 5.99% for  $TE_{11\delta}$ ,  $TE_{21\delta}$ , and  $TE_{01\delta}$ wave modes, respectively and 43.07%, 35.98%, 34.86% for the  $TM_{01\delta}$ ,  $TM_{11\delta}$ , and  $TM_{21\delta}$ wave modes, respectively. These results can be applicable for wave mode selection. The anisotropic permittivity in  $\phi$ -direction has no effect in lowering the TE and TM wave mode resonant frequencies.

*Keywords*: anisotropic permittivity, artificial dielectric material (ADM); circular dielectric resonator (CDR), resonant frequency, transverse electric (TE), transverse magnetic (TM).

## 1. Introduction

Artificial dielectric material (ADM) is a dielectric material which has constitutive parameter values different to those of the conventional dielectric material. The ADM is makeable from conventional dielectric material by an electromagnetics process. From the fact when electromagnetic source is fed into conductor and dielectric material, they will give response differently. The electrons move freely from one position to other positions in conductor, in other hand the electrons movement will be bounded in dielectric material. This phenomenon is known as polarization. Even though electrons can move freely in conductor, however if the size of conductor is gradually minimized into finite metal strips, the electrons will certainly move no freely like the movement in dielectric material.

Basically, the structure of ADM consists of metal strip layers etched on conventional dielectric material. The presence of these metal strips causes the change of electromagnetics characteristics in the dielectric layer. It is well known that the electromagnetics characteristics are determined by shape, size, density, arrangements, and spacing of these metal strips [1]. It is

even possible to configure ADM to have negative permittivity [2], negative permeability [3], both of negative permittivity and permeability [4], refractive index less than unity [5], huge permittivity [6], and anisotropic permittivity [7]-[9]. By exploiting these properties, various applications of ADM in microwave devices have been proposed by many researchers such as antenna, filter, absorber, and resonator. Some unique properties of ADM have been explored in conjunction with its application such as to reject unwanted wave modes in the filter [10], to suppress spurious frequencies in the filters [11], to improve the tilted beam of the planar end-fire antennas in elevation plane [12], to reduce the dimension of microwave resonators [13], to minimize reflected electric field magnitude in absorber [14] and to enhance the gain of end-fire bow-tie antenna [15].

One challenge in ADM researches is to make dielectric material which has relative permittivity value in one direction is different with other directions. In this case, the permittivity has magnitude and vector which is known as anisotropic permittivity. Some related researches of ADM with anisotropic permittivity and its applications have been reported in the last decade [6]-[9]. Those researches have been carried out using the simple rectangular shapes which corresponding to analysis on Cartesian coordinate system. It is very rare to find literatures that discuss on circular metal shapes which corresponding to analysis on cylindrical coordinate system. On other hand, there is a need of this kind of system for many applications such as circular waveguide, circular cavity resonator, circular microstrip antenna, coaxial cable and fiber optic. All of the applications need the ADM with anisotropic permittivity which the material property is constructed from circular shapes of metal layers.

In fulfilling the gap above, in this paper a theoretical analysis for characteristic of ADM in a circular shaped is proposed based on cylindrical coordinate system. The analysis is focused on resonant frequency for the ADM with anisotropic permittivity applied as a resonator. Hence for being analyzable, the resonator made of anisotropic ADM is encapsulated in a circular waveguide which is shorted at the both ends. Later, this resonator is called as artificial circular dielectric resonator (CDR). The resonant frequency expressions of artificial CDR are formulated for TE and TM wave modes by using proper boundary conditions required for the Maxwell equations derivation. The anisotropic permittivity of artificial CDR is obtained by making relative permittivity in one direction higher than others and then the resonant frequencies of first three successive TE and TM wave modes are analyzed [16]-[17]. The analysis of resonant frequency is also conducted for conventional CDR loaded in the same waveguide. It is noted the term of conventional CDR represents a resonator which is constructed by natural dielectric materials with isotropic permittivity. The attempts are to demonstrate the impact of each anisotropic permittivity in lowering resonant frequencies. Here, the anisotropic permittivity is predicted able to reduce the resonant frequencies of conventional CDR on those wave modes.

The theoretical analysis of resonant frequency on TE and TM wave modes for artificial CDR are carried out mathematically by using formulation derived from Maxwell equations. The formulation is determined to calculate resonant frequencies on desired TE and TM wave modes as a function of the thickness of ADM with anisotropic and the value of anisotropic permittivity. In the case of the conventional CDR, the resonant frequencies are impacted by the natural dielectric material thickness and isotropic permittivity value denoted by  $\varepsilon_r$ . By using the formulation, an impact of each anisotropic permittivity on desired TE and TM wave modes are investigated. The results are important for resonant wave mode selection in microwave device applications.

#### 2. Related Work

A closest related research topic is the research of ADM in which the permittivity is defined on each direction of Cartesian coordinate system. In principle, the ADM with anisotropic permittivities based on Cartesian coordinate system has three relative permittivities of  $\mathcal{E}_x, \mathcal{E}_y$ 

and  $\mathcal{E}_z$  in x-, y-, and z-direction, respectively. This kind of anisotropic permittivities for ADM

has been investigated numerically and experimentally [6]-[9]. The materials were appropriate to be implemented in microwave fields which have the geometry structure in rectangular shaped such as rectangular waveguide and a rectangular microstrip antenna. It was shown that the rectangular ADM had the relative permittivity in y-direction higher than others, i.e. x- and z-directions. In the investigation, the proposed ADM with anisotropic permittivity was applied as resonator and analyzed by encapsulating in a rectangular waveguide.

Furthermore, the rectangular resonator with anisotropic permittivity was achieved by employing metal strip layers deployed on dielectric substrates. The gap between metal strip layers was deeply explored since it provided mutual coupling in which this is one of the parameters emphasized in the investigation. The resonant frequencies of proposed rectangular resonator were performed through simulation for different resonator thickness. The purpose of analysis was to show the impact of anisotropic permittivity in *y*-direction to the resonant frequencies lowering. While on the experimental steps, the rectangular resonator was fabricated for characteristics measurement and implemented as a waveguide bandpass filter (BPF). They reported that the realized BPF had insertion loss value higher than 0.5dB with good spurious characteristics [7]-[8].

#### 3. Overview of Artificial Circular Dielectric Resonator

For analyzing the impact of anisotropic permittivity to resonant frequency, the ADM as resonator application, i.e. artificial CDR, is loaded in a short-ended circular waveguide. Figure 1 illustrates the artificial CDR with the thickness *d* encapsulated in a circular waveguide with the radius of *a* for the resonant frequency analysis in TE and TM wave modes. The artificial CDR is assumed to be homogeneous and lossless. Meanwhile, the constitutive parameters of artificial CDR are given in (1) in which the permeability equals to permeability of free space and the permittivity has the form of matrix  $[\varepsilon]$  expresses the anisotropic permittivity in cylindrical coordinate system.

$$\mu = \mu_0, \quad [\varepsilon] = \varepsilon_0 \begin{bmatrix} \varepsilon_\rho & 0 & 0\\ 0 & \varepsilon_\phi & 0\\ 0 & 0 & \varepsilon_z \end{bmatrix}$$
(1)

where  $\mu_0$  is the permeability of free space and  $\varepsilon_0$  is the permittivity of free space, whereas  $\varepsilon_{\rho}$ ,  $\varepsilon_{\phi}$ , and  $\varepsilon_z$  are the relative permittivity in  $\rho$ -,  $\phi$ -, and z-direction, respectively.



Figure 1. Illustration of artificial CDR encapsulated in a circular waveguide for resonant frequency analysis based on cylindrical coordinate system.

For simplicity reason in analysis, non-diagonal elements of permittivity matrix are set to be zero, while the diagonal elements are non zero. The fields that exist in the artificial CDR are given by the Maxwell equations in differential forms for the time-harmonic electric fields (E) and magnetic fields (H) as expressed in (2) and (3), respectively.

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$$\nabla \times \vec{E} = -j\omega\mu_0 \vec{H}$$
(2)  
$$\nabla \times \vec{H} = j\omega[\vec{E}]\vec{E}$$
(3)

In the cylindrical coordinate system, the fields assumed to propagate through the circular waveguide in z-direction penetrate the artificial CDR in the TE or TM wave mode as the function of  $e^{j(\omega t - \beta z)}$  with  $\beta$  is the phase constant. In the case of TE wave mode, the component of E in a propagation direction is zero ( $E_z = 0$ ), while for TM wave mode the component of H in a propagation direction is zero ( $H_z = 0$ ). By using some mathematical manipulations for curl function of (2) and (3)in cylindrical coordinate system, the Helmholtz equations for TE and TM wave modes are determined as written in (4) and (5), respectively.

$$\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial H_z}{\partial\rho}\right) + \frac{1}{\rho^2}\left(\frac{k_{\phi}}{k_{\rho}}\right)^2\frac{\partial^2 H_z}{\partial\phi^2} = -k_{\phi}^2 H_z \tag{4}$$

$$\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial E_z}{\partial\rho}\right) + \frac{1}{\rho^2}\left(\frac{k_\rho\sqrt{\varepsilon_\phi}}{k_\phi\sqrt{\varepsilon_\rho}}\right)^2\frac{\partial^2 E_z}{\partial\phi^2} = -k_\rho^2\frac{\varepsilon_z}{\varepsilon_\rho}E_z$$
(5)

From (4) and (5), there are 2 wave number equations for the artificial CDR, i.e.  $k_{\rho}$  and  $k_{\phi}$ , which are given by (6) and (7), respectively.

$$k_{\rho}^{2} = \omega^{2} \mu_{0} \varepsilon_{0} \varepsilon_{\rho} - \beta^{2}$$
<sup>(6)</sup>

$$k_{\phi}^{2} = \omega^{2} \mu_{0} \varepsilon_{0} \varepsilon_{\phi} - \beta^{2}$$
<sup>(7)</sup>

The solution for (4) and (5) can be obtained by using the method of variable splitting in cylindrical coordinate system as expressed in (8).

$$G = R(\rho)\Phi(\phi) \tag{8}$$

where G is defined as  $H_z$  and  $E_z$  for TE and TM wave mode, respectively. According to (8), the Helmholtz equations in (4) and (5) can be split into 2 equations as function of  $\rho$  and  $\phi$  which have the general solution as expressed in (9) and (10), respectively.

$$R(\rho) = A_1 J_n(k_{\phi} \rho) + A_2 N_n(k_{\phi} \rho)$$
<sup>(9)</sup>

$$\boldsymbol{\Phi}(\boldsymbol{\phi}) = A_3 \sin\left(\frac{k_{\rho}}{k_{\phi}} n \boldsymbol{\phi}\right) + A_4 \cos\left(\frac{k_{\rho}}{k_{\phi}} n \boldsymbol{\phi}\right) \tag{10}$$

where  $J_n$  and  $N_n$  are the Bessel and Neumann functions, respectively. Here,  $R(\rho)$  is taken in the Bessel function only, whilst  $\Phi(\phi)$  is solved in cosine function only. Then, by substituting (9) and (10) into (4) and (5), respectively, and applying proper boundary conditions in which  $R(\rho)$  is defined for  $\rho < a$  and  $\Phi(\phi)$  is suitable for the defined value of  $R(\rho)$ , the solution of  $H_z$  and  $E_z$  in time domain can be obtained as expressed in (11) and (12), respectively.

$$H_{z} = H_{0z} J_{n'}(k_{\phi} \rho) cos\left(\frac{k_{\rho}}{k_{\phi}} n\phi\right) e^{j(\omega t - \beta z)}$$
(11)

$$E_{z} = E_{0z} J_{n} \left( k_{\rho} \sqrt{\frac{\varepsilon_{z}}{\varepsilon_{\rho}}} \rho \right) \cos \left( \frac{k_{\phi}}{k_{\rho}} \sqrt{\frac{\varepsilon_{\rho}}{\varepsilon_{\phi}}} n \phi \right) e^{j(\omega t - \beta z)}$$
(12)

Since  $H_z$  in (11) must vanish when  $\rho = a$ , this condition will satisfy when  $J_n'(k_{\phi}\rho) = 0$ . By the condition, it is necessary to choose the value of  $k_{\phi}\rho$  in such a manner that  $J_n'(k_{\phi}\rho) = 0$ . If the  $k_{\phi} = \frac{X'_{np\delta}}{a}$  then  $X'_{np\delta} = k_{\phi}a$  and  $X'_{np\delta}$  is the root of the first derivation of the Bessel's function. For the first three TE wave modes, the values are 1.841, 3.054, and 3.832 [18]. For a conventional CDR,  $k_{\phi}$  is set to be  $k = \frac{X'_{np\delta}}{a}$ . The k is a wave number of a conventional CDR.

By differentiating (11) and (12) respect to  $\rho$ , the component of E in  $\phi$ -direction is obtained expressed in (13). Then by differentiating the results, i.e. (13), respect to z, the component of H in  $\rho$ -direction is determined as written in (14).

$$E_{\phi} = j \frac{\omega \mu_0}{k_{\phi}} H_{0z} \cos\left(\frac{k_{\rho}}{k_{\phi}} n\phi\right) J'_n \left(k_{\phi} \rho\right) e^{j(\omega t - \beta z)}$$
(13)

$$H_{\rho} = -j\frac{\beta}{k_{\phi}}H_{0z}\cos\left(\frac{k_{\rho}}{k_{\phi}}n\phi\right)J'_{n}\left(k_{\phi}\rho\right)e^{j(\omega t - \beta z)}$$
(14)

From (13) and (14), the wave impedance of artificial CDR can be obtained from ratio between the component of E in  $\phi$ -direction and the component of H in  $\rho$ -direction. Hence, the wave impedances are expressed in (15) and (16) for TE and TM wave mode, respectively.

$$Z_{g-in-TE} = \frac{\omega\mu_0}{\beta} \tag{15}$$

$$Z_{g-out-TM} = \frac{\beta}{\omega \varepsilon_0 \varepsilon_\rho} \tag{16}$$

Since the artificial CDR with the thickness of *d* is placed in the middle of circular waveguide, so there are free space regions in the front and rear sides. In this case, an evanescent mode occurs in the free space regions with the phase constant ( $\beta$ ) in (15) and (16) defined as  $\beta = -j\alpha$  and the relative permittivity in  $\rho$ -direction ( $\varepsilon_{\rho}$ ) set to be 1. As a consequence, the wave number at the free space regions is written in (17).

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$$\left(\frac{X'_{np}}{a}\right)^2 = \left(\frac{\omega}{c}\right)^2 + \alpha^2 \tag{17}$$

Moreover by employing the short-open termination method [8], the input impedance of artificial CDR can be calculated as given in (18).

$$Z_{in} = jZ_g \tan\left(\beta \frac{d}{2}\right) \tag{18}$$

When the magnitude of input impedance equals to the wave impedance at the outside of artificial CDR, it resonates in the certain frequency of each TE mode. In such condition, the expression of relation for TE wave mode resonant frequency and thickness of artificial CDR can be obtained as obtained as written in (19) with c denotes the speed of light in free space and the value of  $\alpha$  and  $\beta$  given in (20) and (21), respectively.

$$\beta \tan\left(\frac{d}{2}\sqrt{\left(\frac{\omega}{c}\sqrt{\varepsilon_{\phi}}\right)^{2} - \left(\frac{X'_{np}}{a}\right)^{2}}\right) = \alpha$$
(19)

$$\beta = \sqrt{\left(\frac{\omega}{c}\sqrt{\varepsilon_{\rho}}\right)^2 - \left(\frac{X'_{np}}{a}\right)^2}$$
(20)

$$\alpha = \sqrt{\left(\frac{\omega}{c}\right)^2 - \left(\frac{X'_{np}}{a}\right)^2} \tag{21}$$

From (19)-(21), the resonant frequency for conventional CDR with the thickness of d which is loaded in the same dimension of circular waveguide can also be calculated by replacing the values of  $\varepsilon_{\rho}$  and  $\varepsilon_{\phi}$  using of  $\varepsilon_r$ . The calculation of resonant frequency for conventional CDR is required for comparison to show the unique features of artificial CDR with anisotropic permittivity.

By using the same procedure as derived for TE wave mode, the expression of relation for TM wave mode resonant frequency and thickness of artificial CDR can also be determined as expressed in (22) with  $\alpha$  and  $\beta$  determined in (21) and (23), respectively.

$$\frac{\beta}{\varepsilon_{\phi}} \tan\left(\beta \frac{d}{2} - \frac{s\pi}{2}\right) = \alpha \tag{22}$$

$$\beta = \sqrt{\left(\frac{\omega}{c}\sqrt{\varepsilon_{\rho}}\right)^{2} - \left(\frac{X_{np}}{a}\sqrt{\frac{\varepsilon_{\rho}}{\varepsilon_{z}}}\right)^{2}}$$
(23)

Similar to the TE wave mode, to calculate the resonant frequency for conventional CDR in the TM wave mode, the values of  $\varepsilon_{\rho}$ ,  $\varepsilon_{\phi}$  and  $\varepsilon_z$  in (22) and (23) are replaced by  $\varepsilon_r$ . It is noted that  $X_{np}$  is the root of Bessel function in which for the first of 3 successive TM wave modes the values are 2.405, 3.832, and 5.136 [18].

#### 4. Calculation, Analysis, and Discussion

Based on the derived mathematical formulation in the previous section, the calculation of resonant frequency for artificial CDR with anisotropic permittivity encapsulated in a circular waveguide is performed. The thickness of artificial CDR is varied from 0.2mm to 4 mm. Meanwhile, the radius of circular waveguide is defined to be 16.27mm with the length set to be long enough compared to the maximum thickness of artificial CDR. The relative permittivity value of artificial CDR for the TE wave mode is examined for 2 different variations, i.e.  $\varepsilon_{\rho} = 10, \varepsilon_{\phi} = 5$  and  $\varepsilon_{\rho} = 5, \varepsilon_{\phi} = 10$ . As comparison, the calculation of TE wave mode resonant frequency for conventional CDR encapsulated in the same dimension of circular waveguide is also performed for isotropic permittivity ( $\varepsilon_r$ ) of 5 with the same variation of thickness.

Figures 2 and 3 plot the numerical calculation results of resonant frequencies obtained using (19)-(21) for anisotropic artificial CDR with 2 different variations of relative permittivity value. The first of 3 successive TE wave modes, i.e.  $\text{TE}_{11\delta}$ ,  $\text{TE}_{21\delta}$ , and  $\text{TE}_{01\delta}$ , for conventional CDR with isotropic permittivity are depicted together as comparison. From Fig. 2, it shows that the resonant frequencies for artificial CDR are lower than of the conventional CDR for the value of  $\varepsilon_{\rho}$  twice than of  $\varepsilon_{r}$ . The different resonant frequencies between artificial CDR and conventional CDR are becoming smaller for higher order wave modes with the maximum

conventional CDR are becoming smaller for higher order wave modes with the maximum value up to 8.5% for TE<sub>11δ</sub> wave mode. The similar trend occurs also for the thickness of artificial CDR in the same wave mode where the thicker thickness of CDR has the smaller different resonant frequencies. This is contradictive with the calculation results shown in Fig. 3 for the value of  $\varepsilon_{\phi}$  twice than of  $\varepsilon_r$  in which there are almost no different resonant frequencies between artificial CDR and conventional CDR.



Figure 2. Resonant frequencies for first of 3 successive TE wave modes ( $\varepsilon_{\rho} = 10, \varepsilon_{\phi} = 5$  for artificial CDR and  $\varepsilon_r = 5$  for conventional CDR).



Figure 3. Resonant frequencies for first of 3 successive TE wave modes ( $\varepsilon_{\rho} = 5, \varepsilon_{\phi} = 10$ for artificial CDR and  $\varepsilon_r = 5$  for conventional CDR).

From those results, it can be inferred that the anisotropic permittivity of artificial CDR in  $\rho$ -direction has more significant impact in resonant frequency lowering for the TE wave mode than the anisotropic permittivity in  $\phi$ -direction. This can be explained from (19)-(20) that the resonant frequency of artificial CDR is inversely proportional to the root of relative permittivity in  $\phi$ - and  $\rho$ -direction. Therefore, the difference value of  $\varepsilon_{\phi}$  and  $\varepsilon_{\rho}$  will give no change remarkably to the resonant frequency of artificial CDR for the TE wave mode.

Different with the TE wave mode, the calculation of TM wave mode resonant frequency for artificial CDR with anisotropic permittivity encapsulated is examined for 3 different variations, i.e.  $\varepsilon_{\rho} = 10, \varepsilon_{\phi} = 5, \varepsilon_z = 5$ ,  $\varepsilon_{\rho} = 5, \varepsilon_{\phi} = 10, \varepsilon_z = 5$ , and  $\varepsilon_{\rho} = 5, \varepsilon_{\phi} = 5, \varepsilon_z = 10$ . The thickness of artificial CDR is also varied from 0.2mm to 4mm with the dimension of circular waveguide is the same as one used in the TE wave mode investigation. The calculation of resonant frequency is conducted for the first of 3 successive TM wave modes, i.e.  $TM_{01\delta}$ ,  $TM_{11\delta}$ , and  $TM_{21\delta}$ . As comparison, the calculation of resonant frequency for conventional CDR in the TM wave mode is also performed for isotropic permittivity ( $\varepsilon_r$ ) of 5 with the same variation of thickness.

The calculation results of resonant frequency for TM<sub>018</sub>, TM<sub>118</sub>, and TM<sub>218</sub> wave modes obtained using (22)-(23) are depicted in Figures 4, 5, and 6, respectively, for 3 different variations of anisotropic permittivity. The calculation results for conventional CDR with isotropic permittivity are also plotted together for each corresponding result as comparison. From the results, it shows that the anisotropic permittivity of artificial CDR in *z*-direction shown in Fig. 6 has the highest contribution in lowering the resonant frequency for TM wave mode up to 72% among to other values of anisotropic permittivity. This means that the component of  $E_z$  has the strongest magnitude compare to the component of  $E_\rho$  and  $E_\phi$  in the circular waveguide. Whilst, the thickness of artificial CDR in the TM wave mode has the similar tendency to the TE wave mode in which the thicker thickness has the smaller different resonant frequencies.



Figure 4. Resonant frequencies for first of 3 successive TM wave modes ( $\varepsilon_{\rho} = 10, \varepsilon_{\phi} = 5, \varepsilon_z = 5$  for artificial CDR and  $\varepsilon_r = 5$  for conventional CDR).



Figure 5. Resonant frequencies for first of 3 successive TM wave modes ( $\varepsilon_{\rho} = 5, \varepsilon_{\phi} = 10, \varepsilon_z = 5$  for artificial CDR and  $\varepsilon_r = 5$  for conventional CDR).

It is evidence that when the anisotropic permittivity of artificial CDR has the same direction with electric field which has the strongest magnitude, it will be dominant in lowering the resonant frequency. Even this fact is reinforced by the anisotropic permittivity in  $\rho$ -direction, since it corresponds to the component of  $E_{\rho}$  which has the magnitude smaller than

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the component of  $E_z$ , the anisotropic permittivity is able to reduce the resonant frequency only for the thickness of artificial CDR less than 2mm as shown in Fig. 4. Moreover, the anisotropic permittivity of artificial CDR in  $\phi$  -direction plotted in Fig. 5 has the lowest impact in lowering the resonant frequency for TM wave mode. This can be understood since the mathematical formulation of resonant frequency for TM wave mode expressed in (22)-(23) has no function of  $\varepsilon_{\phi}$ , therefore the value of  $\varepsilon_{\phi}$  will have no contribution to the resonant frequency lowering of artificial CDR.



Figure 6. Resonant frequencies for first of 3 successive TM wave modes ( $\varepsilon_{\rho} = 5, \varepsilon_{\phi} = 5, \varepsilon_z = 10$  for artificial CDR and  $\varepsilon_r = 5$  for conventional CDR).

#### 5. Conclusions

The resonant frequencies of anisotropic artificial circular dielectric resonator (CDR) encapsulated in a waveguide for the first of 3 successive TE and TM wave modes have been analyzed theoretically. The analysis has been carried out based on mathematical formulation derived from Maxwell time-dependent curl equations for anisotropic artificial CDR loaded in a short-ended circular waveguide. The boundary conditions have been applied properly for the waveguide walls to obtain the mathematical formulation. It has been shown that the artificial CDR with anisotropic permittivity could reduce the resonant frequency of conventional CDR. From the analysis, it can be concluded that the anisotropic permittivity of artificial CDR which has a significant impact in lowering the resonant frequencies for the TM wave mode was the anisotropic permittivity in *z*-direction, while for the TE wave mode it was the anisotropic permittivity in  $\rho$ -direction. In addition, the anisotropic permittivity has no effect in lowering the resonant frequencies of TE and TM wave modes if the anisotropic permittivity has the same direction with the lowest magnitude of electric field.

#### 6. Acknowledgement

This work is partially supported by the Directorate of Research and Community Service, the General Directorate of Research and Development Strengthening, the Ministry of Research, Technology, and Higher Education of the Republic of Indonesia under the research contract FY 2017 No. 025/SP2HL/LT/DRPM/IV/2017.

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