

Sliding Mode Control Based on Synthesis of Approximating State Feedback for Robotic Manipulator

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Abstract: In this paper, three paradigms are used to deal with a robot manipulator control problem. These paradigms are feedback linearization method, approximating control by Taylor truncation, and sliding mode approach. Robotic manipulator is highly nonlinear, highly time-varying, and highly coupled. In robotic manipulator there are many uncertainties such as dynamic parameters (eg., inertia and payload conditions), dynamical effects (e.g., complex nonlinear frictions), and unmodeled dynamics. The classical linear controllers have many difficulties in treating these behaviors. To overcome this problem, sliding mode control (SMC) has been widely used as one of the precise and robust algorithms. Application of traditional SMC in nonlinear system uses exact feedback linearization. Geometric differential theory is used to develop exact linearization transformation of nonlinear dynamical system, by using nonlinear cancellation and state variable transformation. Hence, the controller can be synthesized by using the standard sliding mode for linear system. The main weak point of the exact linearization is that its implementation is difficult. This study presents a synthesis SMC based on approximating state feedback for robotic manipulator control system. This approximating state feedback is derived from exact feedback linearization. Based on approximating state feedback, sliding mode controller is derived. The closed loop stability is evaluated by using the Lyapunov like theory.

Keywords: Robotic manipulator, exact feedback linearization, approximating state feedback, sliding mode, Lyapunov like theory

I. Introduction

Robots are ideal candidates for material handling operations, manufacturing, and measuring devices because of their capacity to pick up, move, and release an object, to manipulate both objects and tools and their capacity to explore the three dimensional space.

Nowadays, robotic manipulator is extensively used in the industrial field. The desire of a high-speed or a high-precision performance for this kind of mechanical systems has led to research into improved control systems. These high performance control systems need, in general, the dynamical model of the robotic manipulator in order to generate the control input (Yurkovich, 1992).

Robotic manipulator is highly nonlinear, highly time-varying, and highly coupled. Moreover, there always exists uncertainty in the system model such as external disturbances, parameter uncertainty, sensor errors and so on, which cause unstable performance in the robotic system (Sadati et al, 2005). Almost all kinds of robust control schemes, including the classical sliding mode control (SMC) (Yong, 1978), have been proposed in the field of robotic control during the past decades. SMC design provides a systematic approach to the problem of maintaining stability in the face of modeling imprecision and uncertainty. Application of traditional SMC in nonlinear system uses exact feedback linearization. The main weak point of the exact linearization is that its implementation is difficult. To overcome this difficulty, this paper presents a synthesis SMC based on approximating state feedback for robotic manipulator control system. This approximating state feedback is derived from exact feedback linearization. The closed loop stability is evaluated by using the Lyapunov theory. During the past decade, several design methods, e.g., robust control (Torres et al, 2007), discrete-time repetitive optimal control (Fateh and Baluchzadeh, 2016) adaptive control (Yazarel and Cheah, 2002; Wang, 2017), backstepping control (Lotfazar et al, 2003; Nikdel et al, 2017), neural network (He et al, 2017; Patino et al, 2002), and fuzzy logic (Kim et al, 2001; Nazemizadeh et al, 2014) for robotic manipulator control have been proposed. In addition, a sliding mode control based on feedback linearization method (Moldoveanu, 2014; Soltanpour and Fateh, 2009) was proposed to control robotic manipulator. A robust control approach is developed to control robot in the task space using sliding mode by support of feedback linearization control and backstepping method.

Feedback linearization is a control design approach for nonlinear systems which attracted lots of research in recent years (Fattah, 2000; Karadogan and Williams II, 2013; Mokhtari et al, 2006; Spong and Groeneveld, 1997). The central idea is to algebraically transform nonlinear systems dynamics into (fully or partially) linear ones, so that linear control techniques can be applied. In the standard approach to exact feedback linearization, one uses coordinate transformation and static state feedback such that the closed-loop system, in the defined region, takes a linear canonical form. After the system's linearization form is obtained, the linear control design scheme is employed to achieve stabilization or tracking (Isidori, 1995; Slotine and Li, 1991).

In the above exact feedback linearization, the controller characteristics have nonlinear functions such as multiplications of the state variables, polynomial functions, trigonometric functions, and so on, which the implementation of the controllers by using electronic devices have many difficulties (Gray and Meyer, 1977; Mahayana, 1991; Nurbambang and Mahayana, 1990; Rangan et al, 1992). On the other hand, many researchers have proven that the performance of the controllers still maintain good response although the approach of the exact feedback linearization have been used (Chong et al, 1991; Koo et al, 2014; Ogawa et al, 1991).

In (Mahayana, 1998; Mahayana, 2011) has been developed a synthesis of nonlinear control system to find a control methodology that makes the exact linearization controller more realizable, but without any significant performance degradation. Instead of the exact controller, the proposed controller was a general form of controller candidates which replace the function of the exact controller. The closed loop stability of the nonlinear system under the controller was evaluated by using the Lyapunov stability theory. The condition under which the origin of the closed loop system being asymptotically stable was derived by characteristic value shift theorem.

In the previous research (Mahayana, 2011) the gravity was ignored, nevertheless the existence of gravity may degrade the control performance. By using approximating state feedback, zero steady state error is not guaranteed. To improve the previous research we synthesize the sliding mode control based on approximating state feedback for robotic control system on two steps. The first is to synthesize the approximating state feedback based on exact feedback linearization. The second is to derive sliding mode control to cope the uncertainty. Simulation results prove the validity of accurate tracking capability and the robust performance.

II. Dynamics of Robotic Manipulator

For simplicity, the robotic manipulator to be controlled just has two joints. The structure of the two-joints manipulator is shown in Figure 1 (Sun and Wang, 2004). In Figure 1, m_1 and m_2 are masses of arm1 and arm2 respectively; l_1 and l_2 are lengths of arm1 and arm2; t_1 and t_2 are torque on arm1 and arm2; θ_1 and θ_2 are positions of arm1 and arm2. The dynamics model of two-link robot can be formulated as

$$M(q)\ddot{q} + B(q,\dot{q})\dot{q} + G(q) = T$$
(1)

where $\boldsymbol{q} = \begin{bmatrix} \theta_1 & \theta_2 \end{bmatrix}^T$ is the joint position vector; $\boldsymbol{M}(\boldsymbol{q}) \in \Re^{n \times n}$ denotes the moment of inertia; $\boldsymbol{B}(\boldsymbol{q}, \dot{\boldsymbol{q}}) \dot{\boldsymbol{q}}$ are the Coriolis and centripetal forces; $\boldsymbol{G}(\boldsymbol{q})$ includes the gravitational forces; $\boldsymbol{T} = \begin{bmatrix} t_1 & t_2 \end{bmatrix}^T$ is the applied torque vector.

Let $c_i \equiv \cos \theta_i$, $c_{ij} \equiv \cos(\theta_i + \theta_j)$, then M, B, G in (1) can be described as

$$M(q) = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix},$$

$$B(q, \dot{q}) = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix},$$

$$G(q) = \begin{bmatrix} (m_1 + m_2)l_1gc_2 + m_2l_2gc_{12} \\ m_2l_2gc_{12} \end{bmatrix}$$
(2)

where

$$m_{11} = (m_1 + m_2) l_1^2 + m_2 l_2^2 + 2 m_2 l_1 l_2 c_2$$

$$m_{12} = m_{21} = m_2 l_2^2 + m_2 l_1 l_2 c_2$$

$$m_{22} = m_2 l_2^2$$

$$b_{11} = -m_2 l_1 l_2 \dot{\theta}_2 \sin \theta_2$$

$$b_{12} = -m_2 l_1 l_2 (\dot{\theta}_1 + \dot{\theta}_2) \sin \theta_2$$

$$b_{21} = m_2 l_1 l_2 \dot{\theta}_1 \sin \theta_2$$

$$b_{22} = 0$$



Figure 1. Structure of Two-Joints Robotic Manipulator

The inertial matrix M(q) is symmetric and positive definite. It is also bounded as a function of \boldsymbol{q} : $\mu_1 \boldsymbol{I} \leq M(\boldsymbol{q}) \leq \mu_2 \boldsymbol{I}$. $\dot{M}(\boldsymbol{q}) - 2\boldsymbol{B}(\boldsymbol{q}, \dot{\boldsymbol{q}})$ is skew symmetric matrix, that is, $\boldsymbol{x}^T [\dot{M}(\boldsymbol{q}) - 2\boldsymbol{B}(\boldsymbol{q}, \dot{\boldsymbol{q}})] \boldsymbol{x} = 0$, where $\boldsymbol{x} \in \Re^{n \times 1}$ is a nonzero vector.

Define $\boldsymbol{u} = \boldsymbol{T}$ and state variable $\boldsymbol{x} = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \end{bmatrix}^T = \begin{bmatrix} \theta_1 & \dot{\theta}_1 & \theta_2 & \dot{\theta}_2 \end{bmatrix}^T$, the equations of motion for the robotic manipulator can be put in the form of following nonlinear state space:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})\mathbf{u} \tag{3}$$

where f(x) is a nonlinear continuous function whose upper bound is known as $|f(x)| \le f_{\max}$, g(x) is a gain function with lower bound g_{\min} , $0 < g_{\min} \le g(x)$.

3. Controller Synthesis

A. Matrix Norm and Spectral Radius

Definition 1. (Goldberg, 1992; Lancaster and Tismenetsky, 1961) If $A \in C^{n \times n}$, then spectral norm of A will be defined as

$$\|\boldsymbol{A}\|_{s} \stackrel{def}{=} \sup_{\boldsymbol{w} \in \boldsymbol{C}^{n}} \frac{\|\boldsymbol{A}\boldsymbol{w}\|_{2}}{\|\boldsymbol{w}\|_{2}}$$

$$(4)$$

Definition 2. (Goldberg, 1992; Lancaster and Tismenetsky, 1961) Spectral radius of a square matrix $A \in C^{n \times n}$, $\rho(A)$, is the maximum among the absolute values of the characteristic values of the matrix A.

To compute spectral norm value of a square matrix, we depart from (Lancaster and Tismenetsky, 1961)

$$\left\|\boldsymbol{A}\right\|_{\mathrm{s}} = \left(\rho(\boldsymbol{A}\boldsymbol{A}^*)\right)^{\frac{1}{2}},\tag{5}$$

where subscript * denotes conjugate transpose of a matrix.

B. Lyapunov Theory and Linearization Consider nonlinear system of the form $\dot{x} = f(x),$ (6)

with f(0) = 0, or in other words the origin of system is the equilibrium point, and f is a continuous vector field and at least once differentiable with respect to x, then the system can be approximated by using a linear time invariant system as follows,

$$\dot{\boldsymbol{x}} = \boldsymbol{\Psi} \boldsymbol{x} \tag{7}$$

$$\Psi \stackrel{def}{=} \frac{\partial f}{\partial x} \Big|_{x=0}$$
(8)

where $\Psi \in \Re^{n \times n}$ is a constant matrix.

Theorem 1. (Khalil, 1992; La Salle and Lefschetz, 1961) If the origin ($\mathbf{x} = \mathbf{0}$) of the linearization result system (7) is asymptotically stable, then the origin of the original system (6) will be asymptotically stable.

C. The method of shifting characteristic values

In this sub-section will be derived the sufficient condition for controller to make the system be asymptotically stable, if the closed loop system under exact controller has an asymptotically stable origin. Let the controller candidate can be expressed as

$$\boldsymbol{u}_{a} = \boldsymbol{u}_{a}(\boldsymbol{x}), \tag{9}$$

and $u_a(x)$ at least once differentiable with respect to x and $u_a(0) = u(0)$.

The error between exact controller and controller candidate is

$$\boldsymbol{e}(\boldsymbol{x}) = \boldsymbol{u}_a(\boldsymbol{x}) - \boldsymbol{u}(\boldsymbol{x}) \tag{10}$$

Assume a notation

$$A_c = A + BK \tag{11}$$

with A_c is closed loop system matrix with exact controller, and also be defined respectively, several variables as follows:

$$v(\boldsymbol{A}_{c}) = \inf_{\boldsymbol{P}} \left(\left\| \boldsymbol{P}^{-1} \right\|_{s} \left\| \boldsymbol{P} \right\|_{s} \right)$$
(12)

$$\boldsymbol{\varepsilon}(\boldsymbol{x}) \stackrel{def}{=} \left(L_{\mathbf{g}} L_{\mathbf{f}}^{n-1} T_{1}(\boldsymbol{x}) \right) \boldsymbol{e}(\boldsymbol{x})$$
(13)

$$\kappa \stackrel{def}{=} \left\| \left(\left(\frac{\partial \varepsilon(\boldsymbol{x})}{\partial \boldsymbol{x}} \right)^T (\nabla \boldsymbol{T}(\boldsymbol{x}))^{-1} \Big|_{\boldsymbol{x}=\boldsymbol{0}} \right)^T \right\|$$
(14)

with P is transformation matrix which transforms A_c to its diagonal canonical form;

$$\boldsymbol{P}\boldsymbol{A}_{\boldsymbol{c}}\boldsymbol{P}^{-1} = diag(\lambda_1, \lambda_1, \cdots, \lambda_n)$$
⁽¹⁵⁾

with $\lambda_1, \lambda_1, \dots, \lambda_n$ are characteristic values of closed loop matrix with exact controller A_c , the shortest distance to imaginary axis is noted by λ_c (by assumption that characteristic values lie in the strict left half of the complex plane).

Since the linear system from exact feedback linearization in the Brunovsky canonical form, then the closed loop system can be made such that all its characteristic values are different. This can be done, e.g., by using pole placement method (Chen, 1970). The assumption of all different characteristic values of A_c is necessary to make the transformation of matrix A_c to diagonal canonical form can be done (Boothby, 1975; Lancaster and Tismenetsky, 1961).

Theorem 2. If

$$Re(\lambda_c) + \kappa v(A_c) < 0$$
(16)

then the origin of the system (3) under control input $u_a(x)$ will be asymptotically stable.

Proof. Theorem 2 will be proved in many stages: construction under exact controller, existence of controller candidate, transformation of system under controller candidate to new state space coordinate, Lyapunov stability analysis, and analysis of shifting characteristic value.

D. Construction under exact controller

Equation (1) can be linearized by choosing T appropriately. Taking T of the form (Slotine and Li, 1991)

$$T = M(q) v + B(q, \dot{q}) \dot{q} + G(q)$$
(17)

where $v \in \Re^{2x1}$ is the new control input, leads to

$$\ddot{\boldsymbol{q}} = \boldsymbol{v} \tag{18}$$

By choosing state variable $\mathbf{x} = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \end{bmatrix}^T = \begin{bmatrix} \theta_1 & \dot{\theta}_1 & \theta_2 & \dot{\theta}_2 \end{bmatrix}^T$, it can be seen that the robotic manipulator dynamics can be expressed in the Brunovsky canonical form as

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \mathbf{v}$$
(19)

Define $z_i = [\theta_i \dot{\theta}_i]^T$, i = 1, 2; equation (19) can be put in the form of following three linear subsystems:

$$\dot{z}_i = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} z_i + \begin{pmatrix} 0 \\ 1 \end{pmatrix} v_i$$
(20)

Letting

$$v_i = -\mathcal{G}_{n,i}^2 \varsigma_i - 2\xi_i \mathcal{G}_{n,i} \dot{\varsigma}_i \tag{21}$$

where $\vartheta_{n,i} \in \Re$ denotes i - th natural frequency, ξ_i denotes i - th damping ratio, and i = 1, 2.

To minimize the ITAE criterion for the step input, one can choose all poles with a damping ratio of $\xi_i = 0.707$ and also can be chosen i - th natural frequency as follows

$$\mathcal{G}_{n,1} = 15$$

 $\mathcal{G}_{n,2} = 16$
(22)

The feedback gain K can be computed as follows:

$$\boldsymbol{K}^{T} = \begin{bmatrix} -225 & 0 \\ -21.21 & 0 \\ 0 & -256 \\ 0 & -22.624 \end{bmatrix}$$
(23)

E. Existence of controller candidate

Since $T: \Omega \to T(\Omega) \subset \mathbb{R}^n$ with an open set Ω on \mathbb{R}^n is a diffeomorphism, then T is smooth. Furthermore u(x) will be smooth. Based on the smoothness of u(x), we can choose a new control input $u_a(x)$ that is continuous and at least once differentiable with respect to x with $u_a(0) = u(0)$, and it satisfies

$$\left\|\boldsymbol{u}_{a}(\boldsymbol{x}) - \boldsymbol{u}(\boldsymbol{x})\right\|_{\infty} \leq \delta \tag{24}$$

with δ is a positive constant, at a range $\nu \subset \Omega \subset \Re^n$ where ν is a bounded closed set.

F. System transformation under controller candidate Nonlinear dynamical system with control input $u_a(x)$ is $\dot{x} = f(x) + g(x)u_a(x)$ (25)

Equation (25) can be arranged as

$$\dot{x} = f(x) + g(x) \{ u(x) + e(x) \}$$
(26)

After some complex calculations, Equation (26) can be written in new state variable, z, as follows

$$\dot{z} = A_c z + \begin{bmatrix} 0\\0\\\vdots\\1 \end{bmatrix} \varepsilon \{T^{-1}(z)\}$$
(27)

The dynamic of system under controller candidate in new state space coordinate can be seen as a nonlinear system that consists of a linear part and a nonlinear perturbation.

Since $u_a(x)$ has been chosen such that $u_a(0) = u(0) = 0$, then e(0) = 0, furthermore at z = 0

$$\boldsymbol{A}_{c}\boldsymbol{z} + \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{0} \\ \vdots \\ \boldsymbol{1} \end{bmatrix} \boldsymbol{\varepsilon} \{\boldsymbol{T}^{-1}(\boldsymbol{z})\} = \boldsymbol{0}$$
(28)

This is shown that the equilibrium point of the system under controller candidate is same with the equilibrium point of the system under the exact controller.

G. Lyapunov Stability Analysis Nonlinear system in Equation (27) can be expressed as $\dot{z} = f_c(z)$ (29)

with

$$\boldsymbol{f}_{c}(\boldsymbol{z}) = \boldsymbol{A}_{c}\boldsymbol{z} + \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{0} \\ \vdots \\ \boldsymbol{1} \end{bmatrix} \boldsymbol{\varepsilon} \{\boldsymbol{T}^{-1}(\boldsymbol{z})\}$$
(30)

Since $f_c(z)$ is smooth in a neighborhood of the origin then there exists $\frac{\partial f_c(z)}{\partial z}$ in a neighborhood of the origin. The origin is an equilibrium point, since based on (28) $f_c(\mathbf{0}) = \mathbf{0}$ (31)

Linearization of (29) in a neighborhood of the origin yields

$$\dot{z} = \frac{\partial f_c(z)}{\partial z} \bigg|_{z=0} \cdot z$$
(32)

(33)

Eq. (32) can be expressed as $\dot{z} = (A_c + D)z$

with

$$\boldsymbol{D} = \begin{bmatrix} \boldsymbol{O}_{(n-1)\times n} \\ \vdots \\ \left[\left(\frac{\partial \boldsymbol{\varepsilon}(\boldsymbol{x})}{\partial \boldsymbol{x}} \right)^T (\nabla \boldsymbol{T}(\boldsymbol{x}))^{-1} \right]_{\boldsymbol{x}=\boldsymbol{0}} \end{bmatrix}$$
(34)

According to the Lyapunov stability theory, if the origin of system (33) is asymptotically stable then the origin of system (29) that is a closed loop system under controller candidate $u_a(x)$, will be asymptotically stable. This can be achieved if all characteristic values of matrix $(A_c + D)$ lie in the strict left half of the complex plane.

H. Analysis of shifting characteristic values

The linear system (33) can be described by the sum of a nominal system matrix, A_c and a perturbation matrix, D. Assume $\zeta_1, \zeta_2, \dots, \zeta_n$ are characteristic values of $(A_c + D)$, by definition of characteristic value (Goldberg, 1992; Lancaster and Tismenetsky, 1961), it can be written as follows

$$(\boldsymbol{A}_{c} + \boldsymbol{D})\boldsymbol{y}_{i} = \boldsymbol{\zeta}_{i}\boldsymbol{y}_{i}, \exists \boldsymbol{y}_{i} \neq \boldsymbol{0}, \, \boldsymbol{y}_{i} \in \boldsymbol{C}^{n}$$

$$(35)$$

Assume $A_c = PA_{c_D}P^{-1}$, with $A_{c_D} = diag(\lambda_1, \lambda_2, \dots, \lambda_n)$. Equation (35) can be written as follows

$$\left(\boldsymbol{A}_{\boldsymbol{c}_{\boldsymbol{D}}} + \boldsymbol{P}^{-1}\boldsymbol{D}\boldsymbol{P}\right)\boldsymbol{r}_{i} = \boldsymbol{\zeta}_{i}\boldsymbol{r}_{i},\tag{36}$$

with $r_i = \mathbf{P}^{-1} y_i \neq 0$. After a little manipulation, it can be found

$$\left(\zeta_{i}\boldsymbol{I}-\boldsymbol{A}_{\boldsymbol{c}_{D}}\right)\boldsymbol{r}_{i}=\boldsymbol{P}^{-1}\boldsymbol{D}\boldsymbol{P}$$
(37)

After some complex calculations we can find

$$\frac{\left\|\left(\zeta_{i}\boldsymbol{I} - \boldsymbol{A}_{\boldsymbol{c}_{\boldsymbol{D}}}\right)\boldsymbol{r}_{i}\right\|_{2}}{\left\|\boldsymbol{r}_{i}\right\|_{2}} \le v(\boldsymbol{A}_{\boldsymbol{c}})\left\|\boldsymbol{D}\right\|_{2}$$
(38)

Since

$$\left\|\boldsymbol{D}\right\|_{2} = \left\{\boldsymbol{\rho}(\boldsymbol{D}\boldsymbol{D}^{*})\right\}^{\frac{1}{2}} = \kappa$$
(39)

then

$$\frac{\left\|\left(\zeta_{i}I - A_{c_{D}}\right)r_{i}\right\|_{2}}{\left\|r_{i}\right\|_{2}} \le v(A_{c})\kappa$$

$$(40)$$

Assume λ_{ci} is characteristic value of matrix A_c which has shortest distance to ζ_i , then

$$\left|\zeta_{i}-\lambda_{ci}\right| \leq \frac{\left\|\left(\zeta_{i}\boldsymbol{I}-\boldsymbol{A}_{c_{D}}\right)\boldsymbol{r}_{i}\right\|_{2}}{\left\|\boldsymbol{r}_{i}\right\|_{2}} \leq \upsilon(\boldsymbol{A}_{c})\kappa$$

$$\tag{41}$$

I. Construction of controller candidate

The parameter values used for the robotic manipulator are $m_1 = 4 kg$, $m_2 = 2 kg$, $l_1 = 1 m$, $l_2 = 0.5 m$, g = 9.8 N / kg.

We propose the controller candidate as $u_a(x) = Lx$ (42)

where $\boldsymbol{L} \in \Re^{2x4}$.

The error between exact controller and approximation controller can be expressed as (43)

 $e(x) = -\alpha(x) - \beta(x)Kz(x) + Lx$

where

$$\begin{aligned} \boldsymbol{\alpha}(\boldsymbol{x}) &= \boldsymbol{B}(\boldsymbol{q}, \dot{\boldsymbol{q}}) \, \dot{\boldsymbol{q}} + \boldsymbol{G}(\boldsymbol{q}) \\ \boldsymbol{\beta}(\boldsymbol{x}) &= \boldsymbol{M}(\boldsymbol{q}) \\ \text{Based on (43) it found} \\ \boldsymbol{\varepsilon}(\boldsymbol{x}) &= \boldsymbol{\beta}^{-1}(\boldsymbol{x}) \boldsymbol{e}(\boldsymbol{x}) \end{aligned} \tag{44}$$

After some complex calculations we can find

$$\boldsymbol{\varepsilon}(\boldsymbol{x}) = \begin{bmatrix} \varepsilon_1(\boldsymbol{x}) \\ \varepsilon_2(\boldsymbol{x}) \end{bmatrix}$$
(45)

where

$$\varepsilon_{1}(\mathbf{x}) = \frac{0.5 + \cos x_{3}}{3 - \cos^{2} x_{3}} \alpha_{2} - \mathbf{K}_{1}\mathbf{x}$$
$$-\frac{0.5}{3 - \cos^{2} x_{3}} (\alpha_{11} + \alpha_{12})$$
$$+\frac{0.5\mathbf{L}_{1}\mathbf{x} - (0.5 + \cos x_{3})\mathbf{L}_{2}\mathbf{x}}{3 - \cos^{2} x_{3}}$$

and

$$\varepsilon_{2}(\mathbf{x}) = \frac{0.5 + \cos x_{3}}{3 - \cos^{2} x_{3}} (\alpha_{11} + \alpha_{12}) - \frac{6.5 + 2\cos x_{3}}{3 - \cos^{2} x_{3}} \alpha_{2} - \mathbf{K}_{2}\mathbf{x} - \frac{(0.5 + \cos x_{3})\mathbf{L}_{1}\mathbf{x}}{3 - \cos^{2} x_{3}} + \frac{(6.5 + 2\cos x_{3})\mathbf{L}_{2}\mathbf{x}}{3 - \cos^{2} x_{3}}$$

with

$$\begin{aligned} \alpha_{11} &= -2x_2x_4\sin x_3 - x_4^2\sin x_3,\\ \alpha_{12} &= 58.8\cos x_3 + 9.8\cos(x_1 + x_3),\\ \alpha_2 &= x_2^2\sin x_3 + 9.8\cos(x_1 + x_3),\\ \mathbf{K}_1 &= \begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} \end{bmatrix},\\ \mathbf{K}_2 &= \begin{bmatrix} K_{21} & K_{22} & K_{23} & K_{24} \end{bmatrix},\\ \mathbf{L}_1 &= \begin{bmatrix} L_{11} & L_{12} & L_{13} & L_{14} \end{bmatrix},\\ \mathbf{L}_2 &= \begin{bmatrix} L_{21} & L_{22} & L_{23} & L_{24} \end{bmatrix}\end{aligned}$$

By differentiating $\varepsilon(x)$ with respect to x, and putting the value x = 0, it can be found

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$$\frac{\partial \varepsilon_{1}}{\partial x_{1}} = 225 + \frac{0.5L_{11} - 1.5L_{21}}{2}$$

$$\frac{\partial \varepsilon_{1}}{\partial x_{2}} = 21.21 + \frac{0.5L_{12} - 1.5L_{22}}{2}$$

$$\frac{\partial \varepsilon_{1}}{\partial x_{3}} = \frac{0.5L_{13}}{3} - \frac{1.5L_{23}}{3}$$

$$\frac{\partial \varepsilon_{1}}{\partial x_{4}} = \frac{0.5L_{14} - 1.5L_{24}}{2}$$

$$\frac{\partial \varepsilon_{2}}{\partial x_{1}} = \frac{8.5L_{21} - 1.5L_{11}}{2}$$

$$\frac{\partial \varepsilon_{2}}{\partial x_{2}} = \frac{8.5L_{22} - 1.5L_{12}}{2}$$

$$\frac{\partial \varepsilon_{2}}{\partial x_{3}} = 256 + \frac{8.5L_{23} - 1.5L_{13}}{2}$$

$$\frac{\partial \varepsilon_{2}}{\partial x_{4}} = 22.624 + \frac{8.5L_{24} - 1.5L_{14}}{2}$$
(46)

If we selected

$$\boldsymbol{L}^{T} = \begin{bmatrix} -1912.5 & -337.5 \\ -180.285 & -31.815 \\ -384 & -128 \\ -33.936 & -11.312 \end{bmatrix}$$
(47)

will imply

$$\frac{\partial \varepsilon}{\partial x}\Big|_{x=0} = 0 \tag{48}$$

and furthermore this will imply

$$\kappa = \left\| \left(\left(\frac{\partial \boldsymbol{\varepsilon}(\boldsymbol{x})}{\partial \boldsymbol{x}} \right)^T \left(\nabla \boldsymbol{T}(\boldsymbol{x}) \right)^{-1} \Big|_{\boldsymbol{x} = \boldsymbol{0}} \right)^T \right\| = 0.$$
(49)

Since $\kappa = 0$, then for this case, Equation (16) is always true, so it can be concluded that the origin of closed loop system (1) under the controller

$$\boldsymbol{u}_{a}(\boldsymbol{x}) = \begin{bmatrix} -1912.5 & -337.5 \\ -180.285 & -31.815 \\ -384 & -128 \\ -33.936 & -11.312 \end{bmatrix}^{\prime} \boldsymbol{x}$$
(50)

is asymptotically stable.

J. Sliding mode control

The main drawback of approximating state feedback controller is the existence of a steady state error, due to the existence of uncertainty. The limitations mentioned above have inspired the idea to derive sliding mode controller based on approximating state feedback.

The first stage in designing a sliding mode controller scheme for robotic manipulator, the goal of the control is to drive the joint position q to the desired position q_d . Let us define $e = q - q_d$ as a tracking error vector. We firstly define the sliding variable as follows:

$$s = \dot{e} + \lambda_1 e + \lambda_2 \int_0^t e \, d\tau \tag{51}$$

where $\lambda_i = diag(\lambda_{i1}, \lambda_{i2})$, in which λ_{i1} and λ_{i2} are chosen to minimize ITAE criterion based on exact feedback linearization.

Based on exact feedback linearization, and to apply feedback gain K in Eq. (23) as follows

$$T_{i} = K_{i2}\dot{e}_{i} + K_{i1}e_{i}$$
(52)

will imply

$$\ddot{e}_{i} + K_{i2}\dot{e}_{i} + K_{i1}e_{i} = 0$$
(53)

Based on approximating feedback linearization, and to apply L in Eq. (47) as follows

$$T_{1} = L_{11}e_{1} + L_{12}\theta_{1} + L_{13}\theta_{2} + L_{14}\theta_{2}$$

$$T_{2} = L_{21}\theta_{1} + L_{22}\dot{\theta}_{1} + L_{23}e_{2} + L_{24}\dot{\theta}_{2}$$
(54)
imply

(55)

will imply

$$\ddot{e}_{i} + K_{i2}\dot{e}_{i} + K_{i1}e_{i} = \Delta_{i}$$

where Δ_i is the error between application of exact feedback linearization and approximating feedback linearization.

The condition for the existence of the sliding mode relates to the stability of the representative point (RP) around the sliding surface. This means, under any circumstances, the RP should stick to the sliding surface. In case of ideal sliding mode motion the sliding surface and its phase velocities should be identically zero i.e.

$$\boldsymbol{s} = \boldsymbol{0} \tag{56}$$

$$\dot{\boldsymbol{s}} = \boldsymbol{0} \tag{57}$$

If system states remain on the sliding surfaces chosen, each tracking error e_i will governed after such finite amount of time by the second-order differential equation $\ddot{e}_i + \lambda_{i1}\dot{e}_i + \lambda_{i2}e_i = 0$. Thus each tracking error e_i will converge asymptotically to 0 as $t \rightarrow \infty$ because λ_{i1} and λ_{i2} are positive constants (Slotine and Li, 1991). But in actual practice it is very difficult to tune to this due to various types of uncertainties, inertia of the physical system, and unrealizability of infinitely fast switching. So for all practical purposes the goal is to find control torque T, such that the system state trajectories are driven to the sliding surfaces.

The second stage of the design procedure involves the selection of the control which will ensure that the chosen sliding mode is attained. For this reason, the problem of determining a control structure and associated gains, which ensure the reaching or hitting of the sliding mode, is called the reachability problem. Let the control torque T can be chosen as follows

$$T_{1} = L_{11}e_{1} + L_{12}\dot{\theta}_{1} + L_{13}\theta_{2} + L_{14}\dot{\theta}_{2} - C_{11}sign(s_{1}) - C_{12}s_{1} T_{2} = L_{21}\theta_{1} + L_{22}\dot{\theta}_{1} + L_{23}e_{2} + L_{24}\dot{\theta}_{2} - C_{21}sign(s_{2}) - C_{22}s_{2}$$
(58)

will imply

$$\ddot{e}_{i} + K_{i1}\dot{e}_{i} + K_{i2}e_{i} = \Delta_{i} - C_{i1}sign(s_{i}) - C_{i2}s_{i}$$
(59)

where C_{i1} and C_{i2} are positive constants. It can be proved that by choosing C_{i1} such that $C_{i1} \ge |\Delta_i|_{bound}$ (60)

where $|\Delta_i|_{\text{bound}}$ is the boundary of $|\Delta_i|$, the overall system is asymptotically stable.

Proof: Consider V in equation (61) as the Lyapunov like function candidate

$$V = \frac{1}{2}s_i^2 \tag{61}$$

For $s_i \neq 0$, V > 0. Now taking the derivative of V with respect to s_i , one can obtain

$$V = s_{i} \dot{s}_{i}$$

= $s_{i} \{\Delta_{i} - C_{i1} sign(s_{i}) - C_{i2} s_{i}\}$
(62)

Using (60), when $s_i > 0$,

$$\Delta_{i} - C_{i1} sign(s_{i}) - C_{i2} s_{i} = \Delta_{i} - C_{i} - C_{i2} |s_{i}|$$
$$\leq C_{i2} |s_{i}| \leq 0$$

and when $s_i < 0$

$$\Delta_{i} - C_{i1} sign(s_{i}) - C_{i2} s_{i} = \Delta_{i} + C_{i} + C_{i2} |s_{i}|$$

$$\geq C_{i2} |s_{i}| \geq 0$$

so that

 $s_i \{\Delta_i - C_{i1} sign(s_i) - C_{i2} s_i\} \le 0$

The Lyapunov like function candidate in (61) is a positive definite and decrescent function, which implies that s_i is driven to zero in finite time. So the overall system is asymptotically stable.

The discontinuous control term in (58) causes high chattering effect which is undesirable in any dynamic system due to its infinite switching frequency. To remedy the control discontinuity in the boundary layer, the signum function $sign(s_i)$ in (58) is replaced by a saturation function of the form (Slotine and Li, 1991):

$$sat(s_{i}) = \begin{cases} sign(s_{i}), & |s_{i}| \ge \Phi_{i} \\ \frac{s_{i}}{\Phi_{i}}, & |s_{i}| < \Phi_{i} \end{cases}$$
(63)

where Φ_i is the boundary layer thickness.

4. Simulation Result And Discussion

In the simulation, the nominal parameters of manipulator are $m_1 = 4 kg$, $m_2 = 2 kg$, $l_1 = 1 \text{ m}$, $l_2 = 0.5 m$, g = 9.8 N/kg. In this example, the manipulator is expected to take a load from position one ($\theta_1 = 0.5$ rad and $\theta_2 = 1$ rad) to position two ($\theta_1 = 1$ rad and $\theta_2 = 2$ rad). In the first stage, the manipulator moves from the initial position to the position 1 along a predefined trajectory during 2 s. It stays there for 1 s to take the load ($m_{load} = 1 kg$) and start to move from position one to position two at t = 3 s. During the second stage, a disturbance ($t_1 = t_1 + 1000 \text{ N}$) is added to link 1 at t = 3.8 s and disappears at t = 4 s. From the above description, there are totally three dynamic changes in the whole process because of the added load, disturbance, and coming back to be normal when the disturbance disappears.

The simulation results are shown in Figure 2 - Figure 6. As is seen in Figure 2 and Figure 3, the joint angles track the desired trajectories and the proposed controller drive the robotic manipulator to its desired positions.



Figure 2. Tracking of joint 1 with the proposed controller.



Figure 3. Tracking of joint 2 with the proposed controller.



Figure 4. Control torque of joint 1 with the proposed controller.



Figure 5. Control torque of joint 2 with the proposed controller.



Figure 6. Tracking error of joint angles with the proposed controller.

5. Conclusions

The proposed method has five stages. First, the controller is synthesized by using exact feedback linearization. Second, the controller is replaced by the controller candidate which is synthesized by approximating an exact feedback controller. Third, stability of the controller candidate is verified by using Lyapunov theory. Fourth, the sliding mode is implemented. Fifth, the controller candidate is implemented by using digital simulation.

The proposed controller can be implemented by using digital controller if the sampling time is small enough. By choosing a reasonable value for discontinuous control term, the control input will not exceed the saturation level. Figure 4 and 5 show that the control input does not exceed the limitation of input (1000 Nm).

The controller benefits from the well-established theory of the sliding mode control and the simple implementation of approximating state feedback controller. The proposed controller can drive the robotic manipulator to its desired positions. Simulation results situation are provided to show the effectiveness of the proposed scheme.

6. References

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