# Optimal Planning of Smart Distribution Network Based on Efficiency Evaluation Using Data Envelopment Analysis

Elnaz Babazadeh[1], Mohammad Reza Jannati Oskuee[2], Jafar Pourmahmoud[3], and Sajad Najafi-Ravadanegh[4]

[1,3]Department of Applied Mathematics, Azarbaijan Shahid Madani University, Tabriz, Iran
[2,4]Smart Distribution Grid Research Lab, E. E. Dept., Azarbaijan Shahid Madani University, Tabriz, Iran

*Abstract*: Optimal planning of electric distribution networks is a complicated multi-objective and multi-constraint problem. This context will be more challenging, regards to the constraints rising from the nature of smart grids. Optimal planning in such a network needs advanced strategies which consider both technical and economical aspects. Distributed generation (DG) and Capacitor banks have attracted a great attention due to their ability in power loss reduction; voltage profile adjustment; voltage stability improvement and emissions reduction. Ever increasing use of electricity imposes a need to enhance the accountability of the power system and improve the system performance parameters. In the literatures, the optimal sizing and siting of several combinations of DGs and/or capacitors have been studied considering different objective functions. The differences are mainly in the problem formulation, methodology and constraints. In this paper, a new approach for optimal allocation of DGs and Capacitor banks, separately and simultaneously, is proposed. The proposed scheme evaluates both the technical and economical aspects with considering several objective functions. The proposed method can easily be expanded with more objectives to cover all the network planners' preferences. The proposed scheme is the combination of an intelligent algorithm and data envelopment analysis (DEA). So, in this paper firstly intelligent algorithm is applied to the sitting and sizing problem, and then the obtained optimal solutions are prioritized by DEA. The significant advantage of using DEA is that, there is no need to impose the decision maker's idea into the model. And ranking is done, based on the efficiencies of the optimal solutions. The most efficient solution is the one which has improved network parameters considerably and has lowest costs. So, using DEA gives a realistic view of solutions and the provided results are for all, not for a specific decision maker. The proposed scheme is applied on a 33-bus radial distribution network and the obtained results are discussed which are satisfactory.

*Keywords*: Optimal planning, Smart distribution grid, Distributed generation, Capacitor bank, Data envelopment analysis

## 1. Introduction

Ever increasing use of electricity imposes a need to enhance the accountability of the power system and improve the system performance parameters. Optimal planning of electric distribution networks is a complicated problem which is usually divided into sub problems. The medium voltage (MV) network planning is one of the major problems in conventional networks [1]. Another sub problem which has attracted a great attention is DG allocation. Several economical, environmental and technical benefits are the essential drivers that increase the contribution of DGs in distribution networks. Reduction of transmission and distribution costs, saving of fossil fuels, reduction of pollution and greenhouse gases as well as technical advantages like loss reduction, peak shaving, voltage profile and load factor improvements and power quality enhancement are the driven forces behind the DGs application [2, 3]. The ultimate goal of distribution system from the planner's point of view is to provide economical and reliable electric power supply to customers. Therefore, the design of power distribution networks is an important issue which with an optimal scheme, reliable power supply can be provided in a low operation costs [4, 5]. In this regards, there are several papers. Optimal

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siting, sizing and timing of MV substations are solved in [6]. A framework for multistage planning of distribution networks using ICA is proposed in [7]. Optimal planning of distribution networks at the presence of DGs is an active research area studied in several papers for instance in [8, 9]. Many interesting methods and algorithms have been proposed to solve the above mentioned problems. The differences are mainly in the problem formulation, methodology and constraints. In [10] some of the algorithms are clearly discussed. For optimal planning of distributed generation in distribution networks, there are several papers which cover a wide range of methods and include the heuristic algorithms, analytical approaches and power flow methods [1, 11-21]. An analytical approach is used in [11, 12]; [13] uses non-linear programming; [14] employs combination of GA and SA; [15, 16] solve the problem with GA; [17] presents tabu search method; [18] submits fuzzy approach for optimization of its algorithm; [1] and [19] apply load flow approaches; [20] uses sequential optimization and [21] uses heuristic approach. In the radial distribution systems, shunt capacitor banks are widely used for different purposes such as voltage improvement, loss reduction, power factor correction and available capacity increase of the feeders [22, 23]. Therefore optimal siting and sizing of capacitors are important in distribution networks [24, 26]. Because of the complexity of the problem, many optimization techniques and algorithms have been applied to optimal capacitor placement problem [27, 28]. As discussed before, given that DG and Capacitor allocation is an optimization problem, each paper is focused on a specific objective function. Also, in some papers, several objectives are considered. To sum up the different objective functions into a single objective function, a common way is to use weighting factors that is used in the most research works. These schemes with single objective function needs to convert all system technical aspect into correct monetary value which somewhat is difficult or also impossible like voltage profile. On the other hand, in single objective some weights determined by the decision makers are needed which are less reliable. Also, from the experiments, the obtained results from optimization of single objectives do not necessarily guarantee the optimal value for the other system parameters. So, maybe with enhancing one parameter, the others get worse. To overcome the mentioned problem, Data Envelopment Analysis (DEA) is proposed to be used. DEA has proven to be an excellent tool in decision support system for measuring efficiency and productivity of a set of decision-making units (DMUs) where multiple inputs convert into multiple outputs [29-38]. Therefore, by using this approach determining the optimum size and site of DGs and capacitors, considering both economic and technical aspects are better suited.

In DEA there is no necessity to have parameters with the same type. Also, with the proposed scheme, several solutions with different characteristics are ranked considering economic and technical aspects and gives varieties to system planners to choose one. On the other hand, in single objectives some weights determined by the decision makers are needed, while in DEA such weights are not required. DEA is an objective methodology which with no knowledge of weights can rank the solutions. With the proposed scheme, well technical and economical solution which means maximum network efficiency is obtainable. Therefore, in this paper, a new method is proposed, in which single-objective functions are used in a combinatory manner. These objectives are not in the same nature. With this method optimal sitting and sizing of DGs and capacitors is available with considering both technical and economical aspects.

## 2. Problem Formulation

In this paper, a methodology to optimize functionality of active distribution networks at the presence of DGs and capacitor banks by minimizing power losses, improving voltage profile, maximizing voltage stability and reducing emissions, individually and simultaneously, considering the installation, operation and maintenance costs of the system is proposed.

*A. Cost Functions*

*A.1. Power Loss Cost*

Compared to transmission systems, distribution networks are well known for a higher R/X ratio and significant voltage drops, which result substantial power and energy losses along MV and LV feeders. As a result, the reduction of electric distribution network losses is one of the major challenges for the utilities [39]. The real power loss of distribution network is given by (1):

$$P_{TL} = \sum_{k=1}^{N-1} R_k I_k^2 \qquad (1)$$

Where $k$ is the line number, $R_k$ is the resistance and $I_k$ is the current of $k$th line, respectively. $P_{TL}$ is the total real power loss of N-bus distribution network. The objective function for loss reduction is formulated as follows:

$$CF_{Loss} = Min(P_{TL}) \qquad (2)$$

Where $CF_{Loss}$ is considered as the first objective function aimed in planning procedure.

*A.2. Voltage Profile*

The objective function to improve and regulate voltage profile [40] is given by (3):

$$CF_{VP} = Min(\sum_{m}^{N}(V_m - V_{Rated})^2) \qquad (3)$$

Where $m$ is the bus number in a N-bus distribution network, $V_m$ is the voltage of bus $m$ and $V_{Rated}$ is the nominal voltage. Where $CF_{VP}$ is considered as the second objective function aimed in planning procedure

*A.3. Voltage Stability Index*

In [41] the voltage stability is defined as the ability of a power system to maintain voltage in acceptable level where the system is able to control both power and voltage when load is increased. Optimal placement of both DGs and/or capacitor banks can affect the voltage stability of the distribution networks. Equation (4) which is described in [42] is used to formulate the voltage stability index as figure 1 and (4).

$$VS_{Index}(R) = |V_S|^4 - 4[P_R X_k - Q_R R_k]^2 - 4|V_S|^2 [P_R R_k - Q_R X_k]^2 \qquad (4)$$

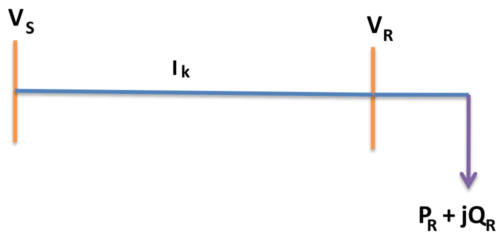


Figure 1. Simple two-node system

Where, $R_k$ and $X_k$ are the resistance and reactance of $k$th line. $P_R$ and $Q_R$ are total real power load and total reactive power load fed through bus $S$. Maximization of voltage stability is formulated as follows:

$$CF_{VS_{Index}} = Max(\frac{1}{VS_{Index}(m)}) \qquad m = 2,3,\ldots,N \qquad (5)$$

In order to maximize voltage stability, weak buses with minimum voltage stability index should be identified and using capacitor or DG the voltage magnitudes are improved at these weak buses. It is obvious that for radial distribution systems to be stable, $VS_{Index}(m) > 0$. Where $CF_{VS_{Index}}$ is considered as the third objective function aimed in planning procedure.

*A.4. Emission*

The minimization of emissions produced by DG units and substation buses such as $NO_x$, $SO_2$ and $CO_2$ results from consumption of fuels can be formulated as [43]. As mentioned, minimizing the emissions is one of the main objective functions in distribution system which can be considered by (6):

$$CF_{Emission} = Min(Emission_{DG} + Emission_{Grid}) \tag{6}$$

Where,

$$\begin{aligned} Emission_{DG} &= P_{DG}(NO_x^{DG} + SO_2^{DG} + CO_2^{DG}) \\ Emission_{Grid} &= P_{Substation}(NO_x^{Grid} + SO_2^{Grid} + CO_2^{Grid}) \end{aligned} \tag{7}$$

In which, $Emission_{DG}$ and $Emission_{Grid}$ are the emission of DG units and substation bus connected to the grid, respectively. The emission factors such as $NO_x$, $SO_2$ and $CO_2$ are produced depending on the fuel; natural gas, oil and liquefied petroleum gas. $P_{DG}$ and $P_{Substation}$ are the produced active power of DG and purchased active power from transmission network, respectively. Where $CF_{Emission}$ is considered as the fourth objective function aimed in planning procedure.

*B. Capacitor Modeling*

The commercially available capacitors have discrete sizes, however they are often assumed as continuous variables. Rounding the continuous optimal values of capacitor sizes, may not guarantee an optimal solution, so it is better to model optimal capacitor placement as an integer-programming problem [24]. In this paper discrete capacitors are considered. In practice, capacitors have standard sizes which are integer multiples of the size $Q_{C0}$. Besides, capacitors of larger size have lower unit prices. The maximum fixed capacitor size is usually limited to:

$$Q_{CMax} = LQ_{C0} \tag{8}$$

Where, $Q_{C0}$ is the base size and $L$ is an integer. The total cost due to capacitor placement is written as:

$$Capacitor\ Cost = \sum (K_{FixCost} + K_{AIC} Q_{Ci}) \tag{9}$$

$K_{FixCost}$ is the fixed cost for the capacitor placement and the $K_{AIC}$ is the annual capacitor installation cost.

*C. Dispatchable DG Modeling*

The output power of a dispatchable DG is considered constant at its rated value and has no associated uncertainties. In the rest of the paper, for simplicity, DG is used instead of dispatchable DG. The available ratings are a discrete multiple of 200kW. The mentioned DG generates both active and reactive power, in which reactive power can be from -0.6 to 0.8 of the active power. In this study, the value of active power; power factor and the site of DG are optimized. Generally, cost per kWh electrical energy produced by DG ( $CF_{P_{DG}}$ ) is a function of the capital cost, fuel cost, and the operation and maintenance costs [44]. The cost function of DG is modeled as follows:

$$DG\,Cost = a + b\ P_{DG} \tag{10}$$

Where a and b are:

$$a = \frac{Capital\cos t(\frac{\$}{kW}) \times Capacity(kW) \times Gr}{LifeTime(Year) \times 365 \times 24 \times LF}$$

$$b = FuelCost(\frac{\$}{kW\,h}) + Op\,Cost\,(\frac{\$}{kW\,h}) + Ma\,Cost\,(\frac{\$}{kW\,h})$$

Load factor is defined by *LF*; annual interest rate by *Gr*; and the cost of operation and maintenance by *Op &Ma* Cost, respectively.

## 3. Proposed Scheme

As discussed before, in this paper, a novel scheme is proposed to find the optimum size and site of several combinations of DGs and capacitors, separately and simultaneously, in a distribution network with considering technical and economical aspects which proper allocation improves the efficiency of the network. In order to find the efficient solution DEA is proposed to be used. DEA is to rank the DMUs from the best to the worst. The best DMU is the one with highest efficiency which in this study means the one which has improved network parameters considerably and has lowest cost. So, in this scheme the efficiency of each DMU has to be estimated. Using the definition of inputs and outputs in DEA, all of the system factors are divided into inputs and outputs. Using an appropriate DEA model, the efficiency of each DMU is determined. Then, in order to rank the efficient DMUs, from the best to the worst, AP model (the super efficiency model proposed by Anderson-Peterson) [40] is employed.

### *A. Data Envelopment Analysis*

Defining the principle of DEA in detail, is out of this paper’s scope and the complete review can be found in several papers, for instance in [29, 30]. A brief description of DEA technique is provided in the following paragraphs. In this paper, DEA is proposed to economically evaluate the system performance. Based on this procedure several single objective functions are considered for optimal sizing and siting of the dispatchable DGs and capacitor banks. DEA is a mathematical programming technique for comparing relative efficiency of a set of firms, usually called as the decision making units (DMUs), which use a variety of identical inputs to produce a variety of identical outputs [29-31]. This method was originated by Charnes, Cooper and Rhodes (CCR) [32] under constant-returns-to-scale (CRS). Banker, Charnes and Cooper (BCC) introduced a variable-returns-to-scale (VRS) version of the CCR model, namely the BCC model [33]. Following the CCR and BCC models, other models of DEA have been introduced in DEA literature. Nowadays DEA is a widely recognized technique and is applicable in several fields of sciences.

Assuming that there are *n* DMUs, each with *m* inputs and *s* outputs, the relative efficiency of a particular $DMU_0 (0 \in \{1,2..n\})$ is obtained by solving the following fractional programming model:

$$Max \quad \theta_0 = \frac{\sum_{r=1}^{s} u_r y_{r0}}{\sum_{i=1}^{m} v_i x_{i0}}$$

$$s.t. \qquad \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \le 1, \qquad j = 1, 2, ..., n$$

$$u_r \ge 0 \quad r = 1, 2, ..., s$$

$$v_i \ge 0 \quad i = 1, 2, ..., m$$

(12)

Where $j$ is the DMU index j=1,2…n, $x_{ij}(i = 1,2..m)$ are the value of $i^{th}$ input for the $j^{th}$ DMU, $y_{rj}(r = 1,2,\ldots,s)$ are the value of $r^{th}$ output for the $j^{th}$ DMU, $u_r(r = 1,2,\ldots,s)$ are the weights of outputs, $v_i(i = 1,2,\ldots,m)$ are the weights of inputs and let the $DMU_0$ be a DMU under evaluation. $DMU_0$ is the efficient if and only if $\theta_0 = 1$. The efficiency of $DMU_0$ is defined as the ratio of its weighted outputs to weighted inputs subjected to the condition that the similar ratios for all DMUs be lower than or equal to 1. A relative efficiency score of 1 indicates that the DMU under consideration is efficient, whereas a score lower than 1 implies that it is inefficient. Model (14) reflects that the goal of DEA is to identify the DMU that has the maximum ratio of its weighted outputs to weighted inputs. As can be seen the weights are determined by the model. So, this technique with no knowledge of weights can rank the DMUs. The objective property of DEA allows us to have a judgment in a just manner. This fractional program can be converted into a linear programming problem: if either the denominator or numerator of the ratio is forced to be unity, then the objective function will become linear, and a linear programming problem can be obtained. By setting the denominator of the ratio equal to 1, the reformulated linear programming problem, also known as the multiplier form of CCR input-oriented model, is as follows:

$$Max \quad \theta_0 = \sum_{r=1}^{s} u_r y_{r0}$$

$$s.t. \quad \sum_{i=1}^{m} v_i x_{i0} = 1$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0 \quad j = 1,2,\ldots,n$$

(13)

$$u_r \ge 0 \quad r = 1,2,\ldots,s$$

$$v_i \ge 0 \quad i = 1,\ldots,m$$

By setting the numerator equal to 1, the multiplier form of CCR output-oriented model is obtained. Because of the nature of the formulations, the optimal objective function values of the CCR input and output oriented represent reciprocal efficiency scores. The dual problem of

(13) is expressed with a real variable $\theta$ and a non-negative vector $\mu = (\mu_1, \mu_2, \ldots, \mu_n)^T$ of variables as follows:

$$\begin{aligned} & Min \ \ \theta \\ & s.t. \quad \sum_{j=1}^{n} \mu_j x_{ij} \le \theta x_{i0} \\ & \sum_{j=1}^{n} \mu_j y_{rj} \ge y_{r0} \\ & \mu_j \ge 0 \quad j = 1,2,\ldots,n \end{aligned} \tag{14}$$

Model (14) is called the envelopment form of CCR-input oriented model. The literature on DEA has many more extensions of these simple models [29-31]. DEA divides DMUs into two groups- efficient and inefficient- while in practice, there is often a need to fully rank them. In DEA models, as usual, more than one DMU may be efficient and their efficiency scores will be 1, so one of the most interesting research matters in DEA is to discriminate efficient units. Some authors have proposed several methods to rank the best performers [34-38]. The super efficiency concept is proposed to differentiate completely among all efficient DMUs when there are more than one efficient DMU. One of the super efficiency models for ranking efficient DMUs in DEA was introduced by Anderson and Peterson [34]. This method enables an extreme efficient unit 0 to achieve an efficiency score greater than one by removing the $0^{th}$ constraint in the envelopment LP formulation, as shown in model

$$\begin{aligned} & Min \ \ \theta \\ & s.t. \quad \sum_{\substack{j=1 \\ j \ne 0}}^{n} \mu_j x_{ij} \le \theta x_{i0} \\ & \sum_{\substack{j=1 \\ j \ne 0}}^{n} \mu_j y_{rj} \ge y_{r0} \\ & \mu_j \ge 0 \quad j = 1, 2, \ldots, n \end{aligned} \tag{15}$$

In this paper DEA is proposed which can be advantageously combined with distribution network planning to determine the well technical and economical scenarios in distribution network. Given that siting and sizing is an optimization problem, Imperialist competitive algorithm (ICA) is found to be an appropriate alternative in this regard. ICA is an optimization algorithm that has recently been developed. ICA optimizes the defined objective functions through the concept of imperialistic competition. Defining the principle of ICA is out of the scope of this paper and the complete review can be found in several papers, for instance [45]. The flowchart of the proposed method based on ICA and DEA is illustrated in Figure 2.

As seen in Figure 2, the initial requirements for the proposed approach is scenario definition with considering network designers' preferences (see 4.3 scenarios 2-7). Here in order to determine the optimal mix of DGs and capacitors, six scenarios are proposed; (scenarios 2-7), along with an extra reference scenario for comparison (scenario 1). For simplicity and limitations in feasible DGs and capacitors location availability, maximum two devices for each technology are considered but it can be expanded to several devices depending on the network capability. In every scenario, minimizing each defined objective function, (power loss, voltage deviation, voltage stability and emission) is to find the optimum site and size of equipment relevant to that objective function. So, considering each scenario, ICA is applied to the defined single objective functions one by one. For each objective function, ICA converge to the optimum solution. The implementation of each solution has its own costs and also affects the technical parameters of distribution network. Until this step, the feasible solutions for optimally allocating of related devices for each scenario are obtained (see Table 2). The number of optimum solutions for each scenario is as the same as the number of objective functions. Afterwards, the optimum solution with its characteristics is called DMU. This principle must be repeated for all the defined scenarios. Finally DEA is applied to the DMUs for efficiency evaluations.

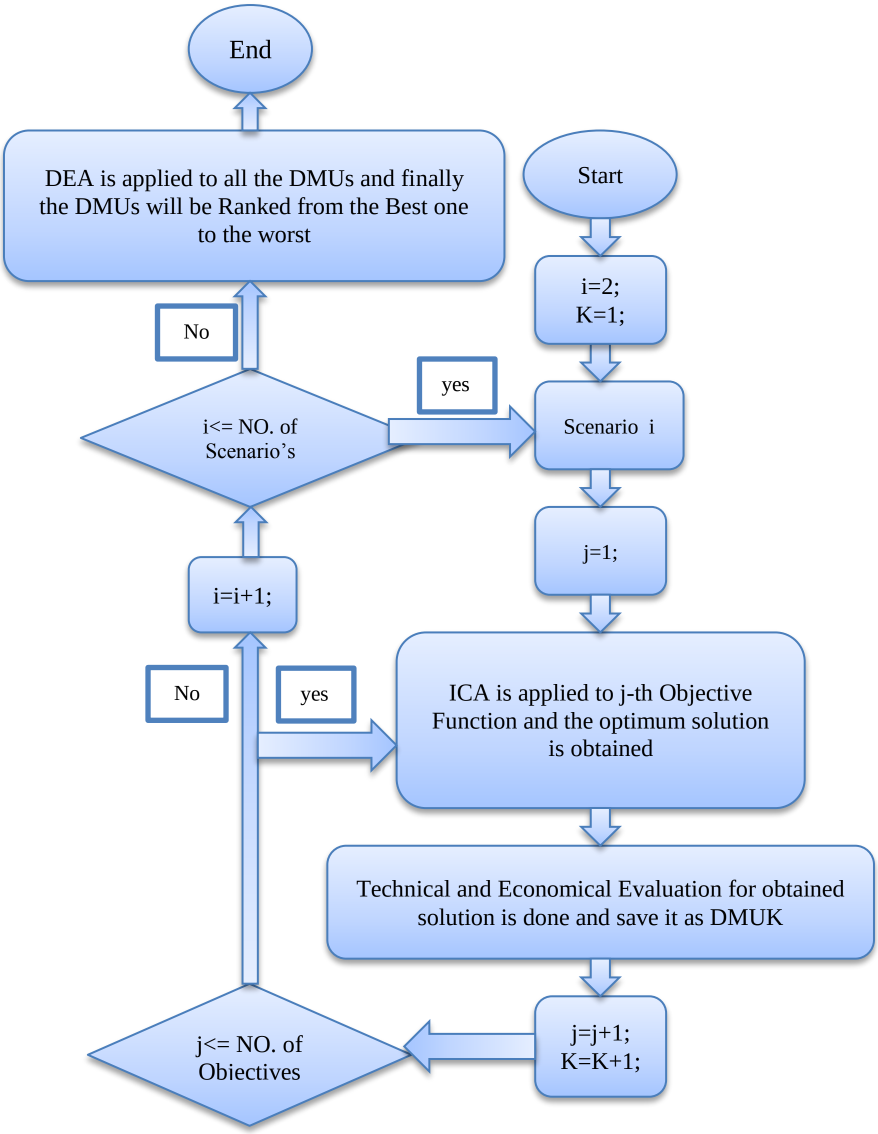


Figure 2. The flowchart of the proposed method based on ICA and DEA

It can be seen from Table 2 that every parameter of system is always minimized when the same objective function is considered, while the other parameters get often worse. So, the results show that optimizing the system for a single objective function does not necessarily guarantee the optimal value for the other system parameters. Also, implementation of each solution needs related investment and operation costs according to the technology used and the system performance characteristics, according to (9) and (10), the cost of purchased energy from transmission system and power loss costs and also the environmental impacts.

For the purpose, DEA is applied because, with this approach determining the optimum size and site considering both economic and technical aspects are better suited.

As discussed before, when several objective functions with different types aimed to be optimized, converting all them into one objective function by using weighting factors, somewhat is difficult and is depended to the decision makers personal preference so it's not common for all. But in DEA there is no necessity to have parameters with the same type. Also, with the proposed scheme, several DMUs (optimum solutions) with different characteristics are ranked considering economic and technical aspects and gives varieties to system planners to choose one. DEA is an objective methodology which with no knowledge of weights can rank the solutions which use a variety of identical inputs to produce a variety of identical outputs. DEA is for comparing the relative efficiency of DMUs with the same multiple inputs and outputs. It means that the relative efficiency measure of each DMU (optimum solution) is obtained without imposing the decision maker's preferences.

## 4. Simulation and Results

*A. System under Study*

The proposed method has been implemented and tested on a 33-bus distribution network. The single line diagram of 33-bus distribution network is shown in Figure 3 [46].

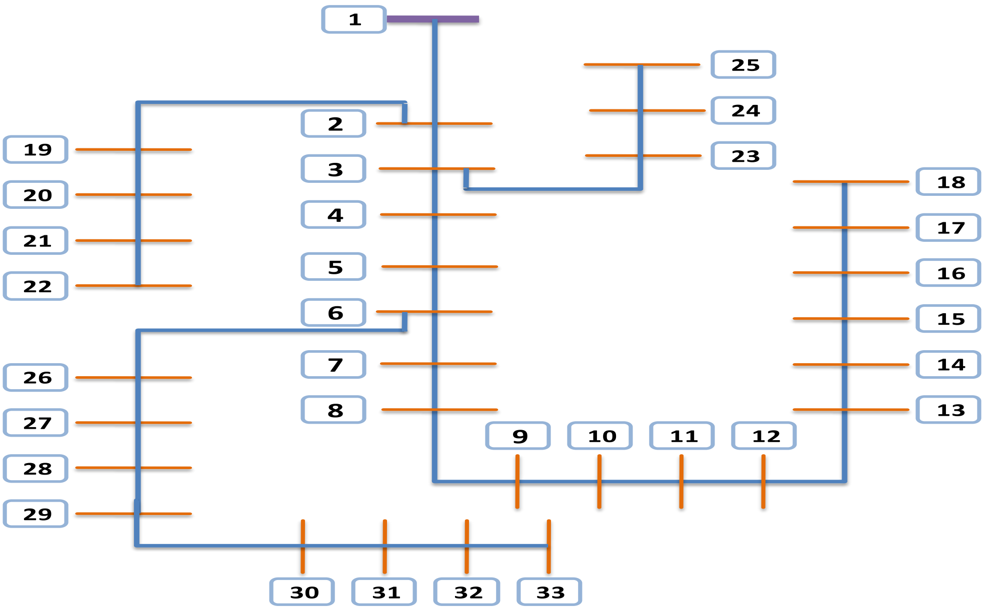


Figure 3. The 33-bus distribution network

In order to determine the optimal mix of DG and capacitors, six scenarios are proposed along with an extra reference scenario for comparison. For simplicity and limitations in feasible DGs location availability, maximum two devices for each technology are considered but it can be expanded to several devices depending on the network capability. The scenarios are as follows:

Scenario #1: the reference scenario, without any DG units or Capacitors (base case);

Scenario #2: Capacitor allocation, one Capacitor can be selected;

Scenario #3: Capacitor allocation, maximum two Capacitors can be selected;

Scenario #4: DG allocation, one DG can be selected;

Scenario #5: DG allocation, maximum two DGs can be selected;
Scenario #6: Simultaneous DG and Capacitor allocation, one DG and one Capacitor can be selected;
Scenario #7: Simultaneous DG and Capacitor allocation, maximum two DGs and two Capacitors can be selected;

*B. Technical Constraints*

Load flow method is based on optimal power flow with its defined constraints which comprises: power flow equations, voltage limit on buses, feeder capacity limit, maximum penetration of DG units in the system and maximum penetration on each bus. Also it should be noted that in each selected bus just one technology can be placed.

*C. Results*

The cost functions are considered as:

$$CF_i = F_i \tag{16}$$

Table 1 shows the single objective function values in scenario #1 as the base case.

Table 1
Cost function (CF) values before allocation

| | |
|---|---|
| $CF_1$ | 224 |
| $CF_2$ | 0.14 |
| $CF_3$ | 1.64 |
| $CF_4$ | 8.26 |

To enhance the performance parameters of the system, considering technical constraints, several scenarios are detailed in Table 2.

Table 2 illustrates the sizing and siting results of DG and capacitors for the rest six scenarios. Second column shows the implemented single objective function for each DMU; Column three depicts the DMU's number, Columns from four to five give the sizing and siting results; and the remained columns show the obtained value of each cost function, given the obtained allocation; namely system parameters. It can be seen from Table 2 that every parameter of system is always minimized when the same objective function is considered, while the other parameters get often worse. So, the results show that optimizing the system for a single objective function does not necessarily guarantee the optimal value for the other system parameters. Also, implementation of each solution needs related investment and operation cost according to the technology used and the system performance characteristics and also the environmental impacts.

With the proposed scheme well technical and economical solution which means maximum network efficiency is obtainable. To this, DEA is applied to Table 2 to find the well technical and economical solution, see 3.1. As seen, third column of table 2 is named DMU and there are 30 DMUs equal to the number of solutions obtained by ICA. As discussed before, each solution is for a specified scenario with relevant objective used. Each DMU defines with the network parameters and has some costs to be implemented.

Table 2. Siting and sizing results of DG and capacitor

| Scenarios | CFs | DMU | Size and site of DGs and Capacitors: DG MW, MVar/ MW,(Bus number) | Size and site of DGs and Capacitors: Capacitor KVar, (Bus number) | $CF_1$ | $CF_2$ | $CF_3$ | $CF_4$ |
|---|---|---|---|---|---|---|---|---|
| # 2 | $CF_1$ | DMU_1 | - | 1200, (30) | 163.34 | 0.10 | 1.59 | 8.13 |
| | $CF_2$ | DMU_2 | - | 1200, (17) | 236.98 | 0.08 | 1.47 | 8.28 |
| | $CF_3$ | DMU_3 | - | 4050, (6) | 243.89 | 0.11 | 1.28 | 8.30 |
| | $CF_4$ | DMU_4 | - | 1200, ( 30) | 163.34 | 0.10 | 1.59 | 8.13 |
| | $CF_5$ | DMU_5 | - | 1200, ( 30) | 163.34 | 0.10 | 1.59 | 8.13 |
| # 3 | $CF_1$ | DMU_6 | - | 1050, (30) 450, (12) | 153.57 | 0.10 | 1.50 | 8.11 |
| | $CF_2$ | DMU_7 | - | 4050, ( 22)1050, (17) | 467.57 | 0.07 | 1.47 | 8.75 |
| | $CF_3$ | DMU_8 | - | 4050, (5) 4050, (6) | 519.59 | 0.19 | 1.08 | 8.86 |
| | $CF_4$ | DMU_9 | - | 1050, (30) 450, (12) | 153.57 | 0.10 | 1.50 | 8.11 |
| | $CF_5$ | DMU_10 | - | 1050, (30) 450, (12) | 153.57 | 0.10 | 1.50 | 8.11 |
| # 4 | $CF_1$ | DMU_11 | 2.6, 0.7, (6) | - | 71.50 | 0.02 | 1.14 | 5.51 |
| | $CF_2$ | DMU_12 | 3.8, 0.5, (7) | - | 95.98 | 0.00 | 1.11 | 4.44 |
| | $CF_3$ | DMU_13 | 5, 0.8, (6) | - | 198.16 | 0.02 | 1.03 | 5.55 |
| | $CF_4$ | DMU_14 | 3.6, 0.8, (8) | - | 215.03 | 0.04 | 1.08 | 4.87 |
| | $CF_5$ | DMU_15 | 3.8, 0.5, (6) | - | 88.27 | 0.00 | 1.07 | 4.43 |
| # 5 | $CF_1$ | DMU_16 | 2.2, 0.8, (3) 1.6, 0.8, (30) | - | 57.53 | 0.02 | 1.23 | 4.36 |
| | $CF_2$ | DMU_17 | 1, 0.8, (10) 1.2, 0.8, (31) | - | 37.10 | 0.00 | 1.10 | 5.82 |
| | $CF_3$ | DMU_18 | 2, 0.6, (8) 2, 0.8, (27) | - | 112.09 | 0.02 | 1.06 | 4.44 |
| | $CF_4$ | DMU_19 | 2, -0.4, (31) 1.6, 0.8, (30) | - | 217.42 | 0.03 | 1.25 | 4.88 |
| | $CF_5$ | DMU_20 | 2.2, 0.8, (3) 1.6, 0.8, (30) | - | 57.53 | 0.02 | 1.23 | 4.36 |
| # 6 | $CF_1$ | DMU_21 | 1.2, 0.4, (10) | 1050, (30) | 70.12 | 0.02 | 1.22 | 6.82 |
| | $CF_2$ | DMU_22 | 1.2, -0.1, (17) | 1200, (31) | 114.54 | 0.00 | 1.19 | 6.91 |
| | $CF_3$ | DMU_23 | 1.2, 0.4, (10) | 1950, (29) | 94.75 | 0.02 | 1.11 | 6.87 |
| | $CF_4$ | DMU_24 | 3.4, -0.4, (12) | 1350, (5) | 419.44 | 0.21 | 1.23 | 5.48 |
| | $CF_5$ | DMU_25 | 1.4, 0.4, (10) | 1050, (30) | 68.31 | 0.01 | 1.20 | 6.63 |
| # 7 | $CF_1$ | DMU_26 | 0.4, 0.6, (14) 0.6, 0.3, (31) | 600, (30) 600, (27) | 57.28 | 0.03 | 1.23 | 6.98 |
| | $CF_2$ | DMU_27 | 1, -0.6, (17) | 1050, (30) 1800, (27) | 149.18 | 0.00 | 1.21 | 7.17 |
| | $CF_3$ | DMU_28 | 1.2, 0.8, (17) | 1650, (31) 1650, (24) | 173.27 | 0.03 | 1.08 | 7.03 |
| | $CF_4$ | DMU_29 | 2.8, 0.3, (31) 0.8, 0.8, (30) | 1050, (27) 1050, (24) | 215.80 | 0.04 | 1.15 | 4.87 |
| | $CF_5$ | DMU_30 | 0.6, 0.2, (31) 0.6, 0.6, (14) | 900, (30) 300, (27) | 46.32 | 0.02 | 1.14 | 6.77 |

## 5. Using DEA Concepts for Assignment of the Efficient Solution

Table 3. DEA and ranking method

| | Results CCR | Results AP | Ranking |
|---|---|---|---|
| DMU_1 | 0.668 | 0.668 | 25 |
| DMU_2 | 0.7182 | 0.7182 | 23 |
| DMU_3 | 0.815 | 0.815 | 21 |
| DMU_4 | | | 25 |
| DMU_5 | | | 25 |
| DMU_6 | 0.7038 | 0.7038 | 24 |
| DMU_7 | 0.7201 | 0.7201 | 22 |
| DMU_8 | 0.9557 | 0.9557 | 12 |
| DMU_9 | 0.7038 | 0.7038 | 24 |
| DMU_10 | | | 24 |
| DMU_11 | 0.9297 | 0.9297 | 14 |
| DMU_12 | 1 | 1.2145 | 2 |
| DMU_13 | 1 | 1.0232 | 4 |
| DMU_14 | 1 | 1.0106 | 6 |
| DMU_15 | 1 | 1.0252 | 3 |
| DMU_16 | 1 | 1.0144 | 5 |
| DMU_17 | 1 | 1.2323 | 1 |
| DMU_18 | 1 | 1.0081 | 7 |
| DMU_19 | 1 | 1.0032 | 8 |
| DMU_20 | | | 5 |
| DMU_21 | 0.8613 | 0.8613 | 19 |
| DMU_22 | 0.9197 | 0.9197 | 15 |
| DMU_23 | 0.9374 | 0.9374 | 13 |
| DMU_24 | 0.9915 | 0.9915 | 10 |
| DMU_25 | 0.8773 | 0.8773 | 18 |
| DMU_26 | 0.8494 | 0.8494 | 20 |
| DMU_27 | 0.9088 | 0.9088 | 17 |
| DMU_28 | 0.959 | 0.959 | 11 |
| DMU_29 | 0.9983 | 0.9983 | 9 |
| DMU_30 | 0.9155 | 0.9155 | 16 |

DEA provides a useful tool in ranking the different solutions obtained for the distribution network. The solutions are the DMUs, as their efficiencies have to be estimated. Using the definition of inputs and outputs in DEA, all of the system factors in this study are considered as inputs. Since DEA calculations cannot be made entirely with inputs and require at least one output, then a dummy output that has a value of 1 for all the DMUs is employed. The terms of

$CF_2$, $CF_3$ and $CF_4$ are considered as inputs. Also the sum of DG Cost and Capacitor Costs and power loss costs is another input. By avoiding the similar DMUs and applying the model (14), we can obtain the following results. As can be seen from Table 3, the CCR-input oriented value for the 8 DMUs is 1 i.e. the 8 DMUs are efficient, so their ranking order is 1. In order to rank the efficient DMUs, AP model (15) is employed. The obtained results using DEA and Ranking Method are detailed in Table 3. The final ranking is presented in the 4th column of Table 3. Note that the vacancies in Table 3 are because some DMUs overlap.

It can be seen from Table 3 that the ranking order of DMU-17 is one, which means that the obtained solution, listed in DMU-17 chart, is the most preferred from the technical and economical points of view. DMU-17 proposes two DGs with $S_1 = 1 + j0.8\ MVA$ and $S_2 = 1.2 + j0.96\ MVA$ at bus numbers of 10 and 31, respectively as indicated in Figure 4. It can be seen that after DMU-17, DMU-12 is preferred and it is followed by DMU-15.

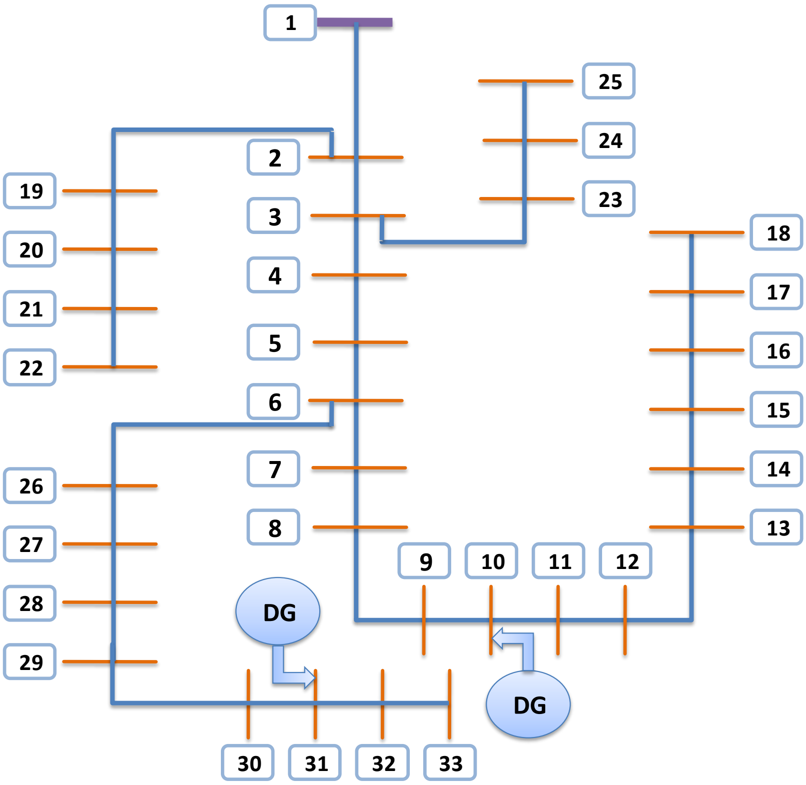

Figure 4. Best technical and economical solution.

It's clear that with the implementing of DMU-17, considerable improvements can be achieved comparing with the base case (scenario 1). The obtained results validate the efficiency of the proposed approach.

## 6. Conclusion

In this paper, a new method is proposed to optimally allocate the DGs and capacitor banks considering both technical and economical aspects. The proposed approach can put into practice considering several objective functions with different types, which in this study are: the network costs minimization, voltage profile adjustment, voltage stability improvement and emission reduction. The proposed method can easily be expanded with more objectives to cover all the network planners' preferences. With the proposed scheme, several solutions with different characteristics are ranked considering economical and technical aspects and gives varieties to system planners to choose one. Also, efficiency evaluation is available with the proposed method to find the well economical and technical solution. On the other hand, with the proposed approach, the most efficient solution for optimal sizing and siting of DGs and capacitor banks is obtained. Simulation results validate the practicability of this method.

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**Elnaz Babazadeh** is a Ph.D student in Azarbaijan Shahid Madani University in Operations Research field. She received her M.Sc. degree from the mentioned university. She has published papers on Data envelopment analysis in Journal of the Operational Research Society (JORS), International Journal of Industrial Mathematics and also different international and foreign conferences. Her major research interests include: Operations Research, Data Envelopment Analysis (DEA), MCDM problems and particularly incorporating DEA with engineering fields such as Electrical Engineering.

**Mohammad Reza Jannati Oskuee** was born in Tabriz, Iran, in 1988. He received his B.Sc. degree in electrical power engineering from University of Tabriz, Tabriz, Iran and M.Sc. degree from Azarbaijan Shahid Madani University, Tabriz, Iran, graduating with first class honors, where he is currently working towards the Ph.D. degree at Electrical Engineering Department. In 2014, he was the recipient of the Best Student Researcher Award of the Azarbaijan Shahid Madani University. His major research interests include: smart grids, power system planning and operation, power electronics, power system dynamics and FACTs devices.

**Jafar Pourmahmoud** is an assistant professor in Department of Applied Mathematics, Azarbaijan Shahid madani University. He has published papers in Bulletin of the Iranian Mathematical Society, Applied Mathematics and computation, Journal of Applied Environmental and Biological Sciences, International Journal of Industrial Mathematics and International Journal of Industrial Engineering Computations. His research interest is on Operations Research (especially on Network Data Envelopment Analysis and fuzzy DEA) and Numerical Linear Algebra and Applications.

**Sajad Najafi Ravadanegh** was born in Iran, in 1976. He received the B.Sc. degree from the University of Tabriz, Tabriz, Iran, and the M.Sc. and Ph.D. degrees from the Amirkabir University of Technology, Tehran, Iran, in 2001, 2003, and 2009, respectively, all in electrical engineering. He is currently an Associate Professor with the Electrical Engineering Department, Azarbaijan Shahid Madani University, Tabriz, where he is responsible for the Smart Distribution Grid Research Laboratory. His current research interests include smart distribution networks and microgrids optimal operation and planning, power system stability and control, power system controlled islanding, optimization algorithms applications in power systems, nonlinear dynamic, and chaos. He has authored/co-authored of over 40 technical papers.