Fractional Wavelet-based QRS Detector

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Abstract: The detection of complex QRS is of a major importance in the systems of automatic treatment of the ECG. Indeed, once the peaks R identified, the heart rate can be calculated and various times and amplitudes of the cardiac cycle can be measured and located. Anomalies can therefore be detected. This paper proposes a new method based on Fractional Wavelet Filters for accurate detection of different QRS in the ECG. This method is based on the different energy levels in Fractional Wavelet detail coefficients and it was tested using ECGs from selected records of the MIT-BIH Arrhythmia Database (MITDB). Results, in terms of error, sensitivity and the value of the positive predictivity are very satisfactory.

Keywords: ECG, QRS detection, Fractional Wavelet, Filters.

1. Introduction

An electrocardiogram (ECG) is a noninvasive test that is used to reflect underlying heart conditions by measuring the electrical activity of the heart [1]. An ECG is thus a plot of the time-dependence of charging potential differences between electrodes on the body surface. A typical ECG is shown in Figure. 1, where the familiar deflections P, Q, R, S, and T are apparent.

Electrocardiography has become one of the most commonly used medical tests in medicine. Electrocardiogram contains an important amount of information that can be exploited in different manners. It allows the diagnosis of a myriad of cardiac pathologies.

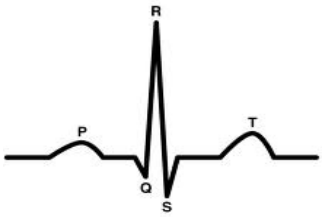

Figure 1. The electrocardiogram (ECG)

The detection of the QRS complex is of a major importance in the systems of automatic treatment of the ECG. It has many applications including R-R interval analysis, ST segment examination, ECG compression and arrhythmia classification. Indeed, once the peaks R identified, it becomes easy to calculate the heart rate and to analyze the variability of heart rate. The difficulties of the QRS detection are, primarily, in the great variability of the form of the signal because their morphology varies from one individual to another, even at the same subject, it varies from one cycle to another. Moreover, noises of various origins, present in the ECG, as well as P and T waves of great amplitudes can also be taken for complexes QRS [2].

In the literature, there is a wide diversity of QRS detection algorithms available which uses a variety of signal analysis methods. A method of detection using adaptive digital filtering is proposed in [3]. This later uses self-adaptive filter in order to maximize the signal-to-ratio for ECG characteristics detection. Authors of [4] have proposed a real-time QRS detection algorithm, which is based on a decision rule process. Similarly, algorithms based on wavelet transforms have been investigated in order to detect ECG characteristics [5-10]. First-derivative-based method is often used in real-time analysis; it aims to maximize detection accuracy instead

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of calculation time [11]. To deal with the nonlinear nature of the ECG, some methods were investigated in the literature using artificial neural network and genetic algorithms [12], [13].

Over the past decade, some research studies have been focused on noise analysis and cancellation; for example, adaptive filters were used in [14] to remove noise; similarly, authors of [15] have proposed a hybrid linearization method that combines extended Kalman filters and discrete wavelet transform in order to de-noise ECG signal. In the same way, authors of [16], [17] have provided an overview and a comparison of recent developments in QRS detection and noise sensitivity analysis. In addition, a PQRST detection algorithm has been proposed, which uses information obtained from 12-lead discrete data [18], [19].

More recently, authors of [20] have proposed an efficient discrete Fourier series-based method to reduce both baseline wander and powerline interference noises in ECG records.

Beyond theoretical aspects, some existing methods were implemented using DSP and microcontroller devices [6], [21], [22].

However, even with such developments, high detection accuracy still remains a challenge. To the best of our knowledge, no one has developed a perfect real-time QRS detection algorithm, since QRS complexes have a time-varying behavior, their detection is sometimes indistinguishable from P and T waves, especially, when the ECG signal is affected with many sources of noise (poor electrode contact, power line interference, muscle contraction, etc...).

In this paper, we propose a new optimum approach for QRS detection based on fractional wavelet which efficiently faces noise. In contrast to traditional wavelet, the main advantage of the use of fractional wavelet is its flexibility in terms of parameters transform adjustment. In this context, two steps are necessary for the implementation of this type of wavelet; the first consists in calculating the transfer function leading to the fractional filters of the wavelet, and the second concerns the combination of these filters with under-sampling operation to have a fractional wavelet. Furthermore, the proposed approach is tested on a standard database including normal and abnormal ECG signals which makes it comparable and easy to evaluate with other approaches reported in the literature for QRS detection [8], [23-26].

2. The proposed approach

A. Wavelet transform

Wavelets have been defined in the 1980s as a multi-scale tool for signal and image analysis. The idea is to choose some intrinsic bases adapted to the representation of a class of signals and then approaching the function with preserving the information as possible. Thus, wavelets are suitable to pattern recognition of medical signals; they are well used for estimating and detecting waves in the context of ECG analysis for the purpose of diagnosis.

The wavelet transform of a signal $f(t)$ is defined in the equation below.

$$Wf(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} f(t)\Psi^*\left(\frac{t-b}{a}\right)dt \tag{1}$$

where Ψ* is the mother wavelet, *a* is the scale factor and *b* is a translation parameter.

A function is said to be a wavelet if it verifies the admissibility condition, i.e., if it has a finite spectrum:

$$\int_{-\infty}^{+\infty} \frac{|\Psi(w)|^2}{|w|} dw = C_w < +\infty \tag{2}$$

C_w is the admissibility constant. This condition implies that its area is zero:

$$\int_{-\infty}^{+\infty} \Psi(t)dt = 0 \tag{3}$$

The choice of a particular wavelet for medical signal analysis is a crucial task. It is worthy to note that there is not a universal method to estimate and detect signal parameters for diagnosis. In fact, the choice of a wavelet depends on the type of analysis and application. There are many classes of wavelets, such as: Haar, Daubechies, Biorthogonal, Coiflets, Symlets, Morlet,

Mexican Hat and Meyer. Haar wavelet has been successfully used by [8]. Wavelet transform based on fractional function is explored by [9], where derivative of Cole-Cole distribution is used to define fractional function.

B. Fractional wavelets

In our work, we present a detection algorithm that uses a fractional wavelet. A model is said to be of fractional order if it is based on a differential equation representation given by:

$$\sum_{l=0}^{L} a_l D^{lq} y(t) = \sum_{m=0}^{M} b_m D^{mq} u(t) \tag{4}$$

where *u(t)* and *y(t)* are the input and output of the system respectively, *D* is the derivative operator, *q* is a rational number, $\{a_l, b_m\} \in \Re$, $0 \le l \le L$ and $0 \le m \le M$.

The Laplace transform of (4) yields to the following transfer function with fractional powers:

$$G(s) = \frac{\sum_{m=0}^{M} b_m s^{mq}}{\sum_{l=0}^{L} a_l s^{lq}} \tag{5}$$

where *q* is a real number, *M* and *L* are two integers with *M*<*L*. The value of *q* is generally set to a rational number 1/*Q*:

$$G(s) = \frac{\sum_{m=0}^{M} b_m s^{m/Q}}{\sum_{l=0}^{L} a_l s^{l/Q}} \tag{6}$$

In general, the rational approximation of the function $G(s) = s^{-n}$, $0 < n < 1$ is obtained using the CFE (continued fraction expansion) [27].

As well known, in wavelet transforms, there are always approximation and detail in the wavelet decomposition. The approximation is the high scale of low frequency components of the signal, while details are low scales of high frequency components. Such filters are expressed as follows:

$$G_h(s) = \frac{1}{(1+sT)^n} \tag{7}$$

$$G_\ell(s) = \left(1 + \frac{1}{s}\right)^n \tag{8}$$

where $G_h(s)$ and $G_\ell(s)$ are respectively the approximation in high and low frequencies.

The filtering process is as follows. The original signal goes through two complementary filters and emerges as two signals. So, the input data is doubled as shown in Figure. 2.

The idea is simple; we can create a fractional wavelet on the basis of fractional order filters. One just has to substitute these ordinary filters for fractional filters. Then, the question of how to express these transfer functions arises.

C. Transfer function

Defining the transfer function is a hard task. We use a certain low-pass transfer function of fractional order [28]:

$$G_\ell(s) = \frac{1}{(1+s)^m} \tag{9}$$

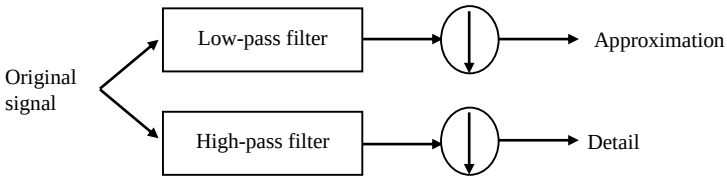


Figure 2. Discrete wavelet transform

where *m* is the order of the system. Every time a pole is added or a degree is changed or the pole is multiplied with a real number that looked for minimizing the error. So, after many experiments the following transfer function is obtained:

$$G_\ell(s) = \frac{1}{1.2 + (0.9s)^{m_1} - (0.4+s)^{m_2} + (0.5+s)^{m_3}} \tag{10}$$

where $m_1 = 2.57$; $m_2 = 11.4$; $m_3 = 5.91$

It is worth to note that the high-pass filter can be directly deduced from the low-pass filter thanks to the following expression:

$$G_h(N+1-i) = G_\ell(-1)^{i+1} i \tag{11}$$

The frequency responses of the both low-pass and high-pass filters and their reconstructions are depicted in Figures. 3a, b and 4a, b, respectively.

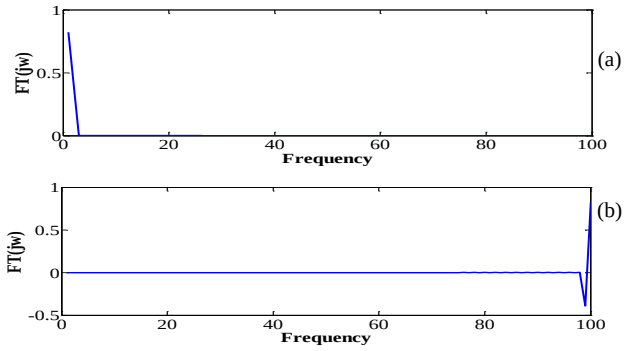


Figure 3. Fractional decomposition filters; (a) Low-pass and (b) High-pass

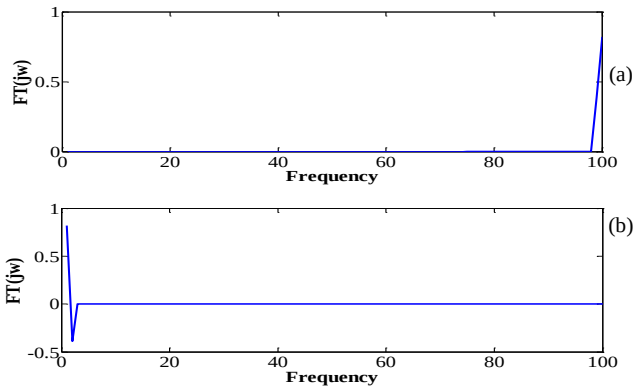


Figure 4. Fractional reconstruction filters; (a) Low- Pass and (b) High-Pass

D. R peak detector

In respect of Nyquist criterion, all the ECGs are sampled at least at 360 Hz. The QRS frequency content is similar in all records of the MITBIH arrhythmia database. The power spectrum is processed by Fast Fourier Transform (FFT) and, for normal ECGs, the energy of the

ECG signal is concentrated within the QRS between 5 Hz and 40 Hz (Figure. 5). As for abnormal ECGs, this interval may exceed 60 Hz [29] (Figure. 5).

In order to make the appearance of the ECG's power spectrum, a successive filtering through low-pass and high-pass filters is performed to many levels. Figure. 6 shows the power spectrum for the 3rd, 4th and the 5th levels as well as the sum $h(i)$. T and P waves have frequencies low than 5 Hz, so they are eliminated. Two kinds of records, namely, normal and abnormal subjects are considered and represented respectively by the numbered records 100 and 208.

The first step in the ECG preprocessing is to perform decomposition. The choice of desired level of decomposition is dependent on required frequency components available in the wavelet coefficient at that level. In our case, using the proposed fractional wavelet, the decomposition is performed up to the 5th level.

For each level, the FFT-based power spectrum is processed. In each segment, the peak of the frequency spectrum obtained corresponds to the energy peak of the QRS complex.

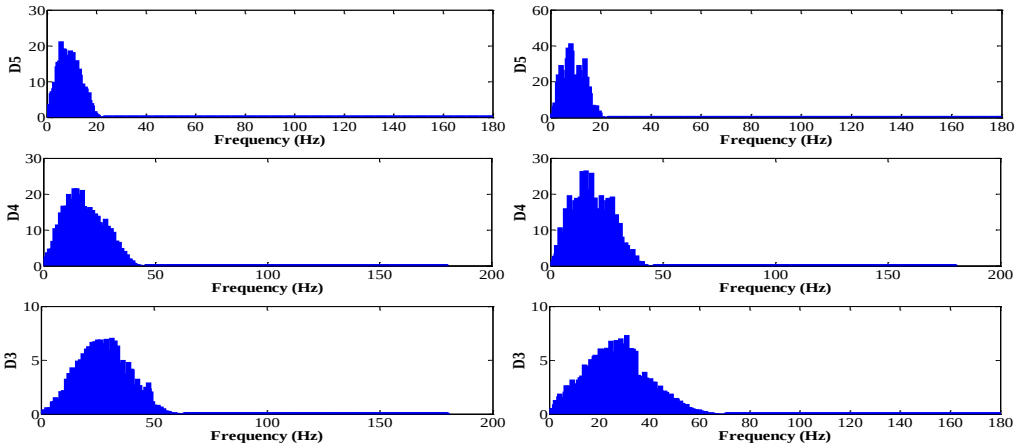
We keep from the record 100 a segment containing two normal beats and from the record 208 a segment with abnormal beats, so the first one shows highest frequencies and the second one shows lowest frequencies (Figure. 5).

In the second time, the square sum h(i) of the selected details is computed (Details D1 and D2 are not represented since they are out of range of R peak frequencies).

$$h(i) = \left(\sum_{j=3}^{5} D_j\right)^2 \tag{12}$$

The procedure used for detecting R peak localization through the proposed fractional wavelet follows the steps given in [8].

- We select $h(i)$ associated to R peak using a threshold $\lambda = 0.1 * \max(h)$.
- if $h(i) \geq \lambda$ we take i as the position of the R peak else it is not R peak position.
- To identify different R peaks: we compute $(i' - i)$ of two consecutive selected positions.
- If $(i' - i) < 36$ then $i,\ i'$ are at the same QRS position, else $i,\ i'$ are not at the same QRS position.



100.dat 208.dat

Figure 5. Power levels in function of the frequency in normal and abnormal ECG.

Elimination of multiple detections: a peak occurring within the refractory period (200 ms) is disregarded. And If no R peak was detected within 150% of the RR interval, then a back search is performed by dividing the threshold by two [4], [11].

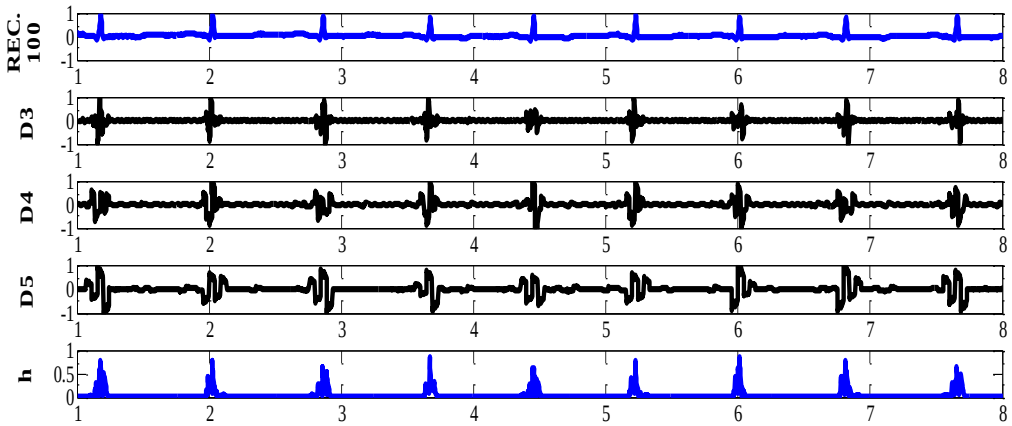


Figure 6. ECG signal Decomposition using fractional wavelet for the record 100.

3. Results and analysis

The ECG records taken from the MITBIH arrhythmia database are sampled at 360 Hz. In accordance with Nyquist's rule, the range of real frequency components of the signals is between 0 and 180 Hz. The algorithm was implemented on a PC with an I7 microprocessor using MATLAB.

The study reports on the analysis of the first minute signal of the 48 records. The R waves detected were compared to the annotation file accompanying each signal to determine the error.

The performance of the R wave detector is evaluated in terms of the number of R waves missed (FN: false negative) and the number of R waves falsely reported (FP: false positive). The error *Er*, or failed detection rate, is defined by:

$$Er=(FP+FN)/TB \tag{13}$$

where *TB* is the total number of beats.

Other statistical parameters are also used to compare the results such as sensitivity S_e and predictivity *P+* which are defined respectively as:

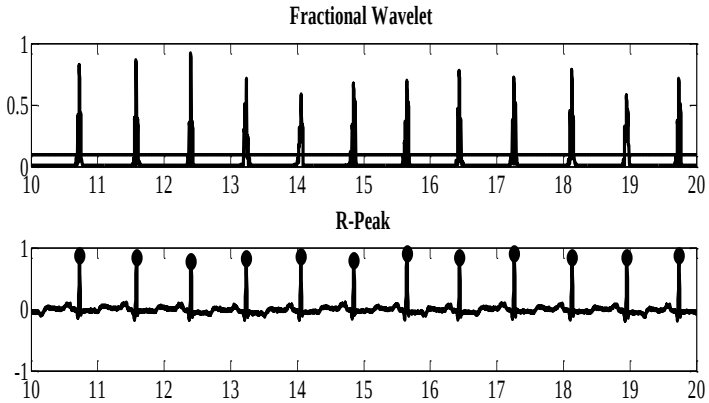
$$S_e = \frac{TP}{TP+FN} \tag{14}$$

$$P+ = \frac{TP}{TP+FP} \tag{15}$$

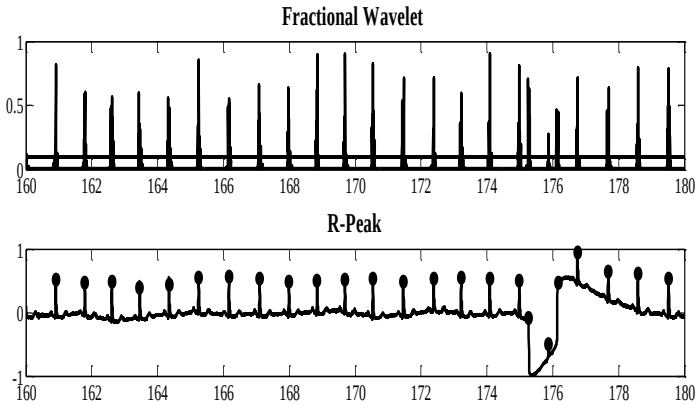
where *TP* stands for True Positive, i.e., the correctly detected beats.

Figures 7 and 8 depict some examples of real ECG signals for the detection of QRS complexes. The indicated cases include examples with different qualities. The 100.dat (Figure7a) is a signal with acceptable quality where R peaks are quite visible with great amplitudes. Its treatment does not pose problem, i.e., all the peaks are detected and the error is 0, 00%. Next, we choose 101.dat (Figure 7b) as a signal with an abrupt jump of the base line.

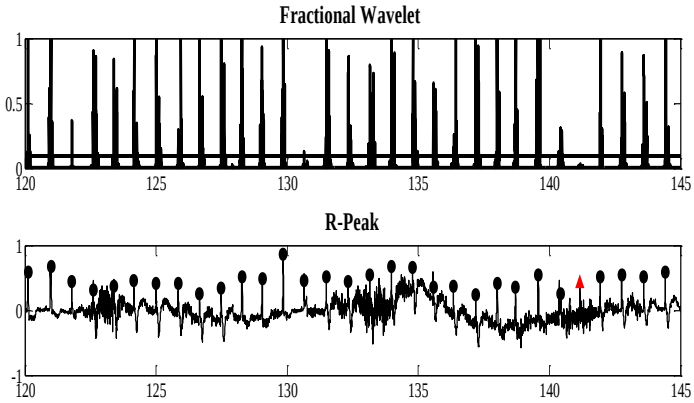
This jump will cause false detections (FP) because of transitions which will be taken for R peaks by the algorithm. In a signal with artifact (104.dat, Figure.7.c), the pretreatment highlights the R peaks in spite of the presence of noise. As for a signal with gradual deviation of the base line (203.dat, Figure.8.a), QRS complexes and full T waves changes do not affect R peaks detection. Finally, for signals with QRS complexes containing two R peaks as in 102.dat and 221.dat (Figure.8.b, Figure.8.c), the algorithm prevailed the peak of greater amplitude; this latter is regarded as a point pertaining to QRS complex.



a. Record 100.dat

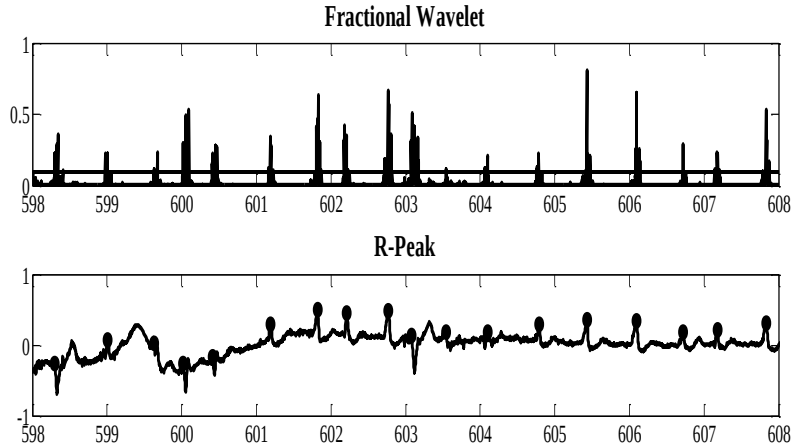


b. Record 101.dat

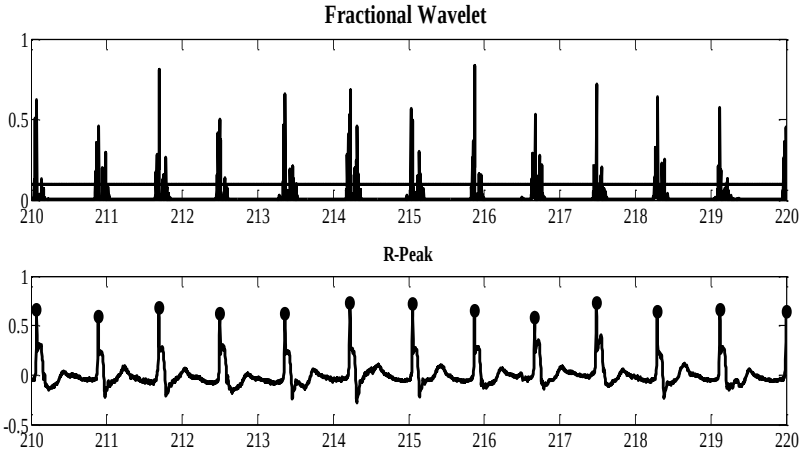


c. Record 104.dat

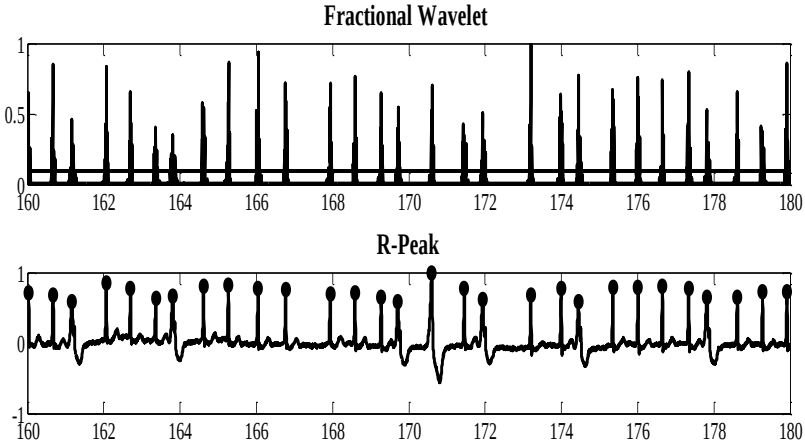
Figures 7. Examples of ECGs for the R wave detector (set 1).



a. Record 203.dat



b. Record 102.dat



c. Record 221.dat

Figures 8. Examples of ECGs for the R wave detector (set 2).

Table 1. Details of the detection of QRS complexes for each signal ECG.

Rec. No	Total QRS	TB	FP	FN	S_e (%)	P+(%)	Er (%)
100	2273	2273	0	0	100,00	100,00	0,00
101	1865	1865	2	0	100,00	99,89	0,11
102	2187	2187	2	0	100,00	99,91	0,09
103	2084	2084	0	0	100,00	100,00	0,00
104	2229	2213	2	16	99,28	99,91	0,81
105	2572	2543	35	29	98,87	98,64	2,49
106	2027	2025	0	2	99,90	100,00	0,10
107	2137	2127	2	10	99,53	99,91	0,56
108	1763	1728	24	35	98,01	98,63	3,35
109	2532	2532	0	0	100,00	100,00	0,00
111	2124	2124	3	0	100,00	99,86	0,14
112	2539	2539	0	0	100,00	100,00	0,00
113	1795	1795	2	0	100,00	99,89	0,11
114	1879	1875	0	4	99,79	100,00	0,21
115	1953	1953	0	0	100,00	100,00	0,00
116	2412	2394	0	18	99,25	100,00	0,75
117	1535	1535	0	0	100,00	100,00	0,00
118	2278	2278	0	0	100,00	100,00	0,00
119	1987	1987	0	0	100,00	100,00	0,00
121	1863	1863	1	0	100,00	99,95	0,05
122	2476	2476	0	0	100,00	100,00	0,00
123	1518	1518	0	0	100,00	100,00	0,00
124	1619	1619	0	0	100,00	100,00	0,00
200	2601	2599	3	2	99,92	99,88	0,19
201	1963	1958	2	5	99,75	99,90	0,36
202	2136	2134	1	2	99,91	99,95	0,14
203	2980	2941	27	39	98,69	99,09	2,21
205	2656	2653	0	3	99,89	100,00	0,11
207	1860	1825	8	35	98,12	99,56	2,31
208	2955	2939	9	16	99,46	99,69	0,85
209	3005	3005	2	0	100,00	99,93	0,07
210	2650	2650	3	2	99,92	99,89	0,19
212	2748	2748	0	0	100,00	100,00	0,00
213	3251	3250	0	1	99,97	100,00	0,03
214	2262	2261	1	1	99,96	99,96	0,09
215	3363	3361	0	2	99,94	100,00	0,06
217	2208	2208	0	0	100,00	100,00	0,00
219	2154	2154	0	0	100,00	100,00	0,00
220	2048	2048	0	0	100,00	100,00	0,00
221	2427	2424	0	3	99,88	100,00	0,12
222	2483	2470	2	13	99,48	99,92	0,60
223	2605	2605	0	0	100,00	100,00	0,00
228	2053	2036	28	17	99,17	98,64	2,19
230	2256	2256	3	0	100,00	99,87	0,13
231	1571	1571	0	0	100,00	100,00	0,00
232	1780	1780	1	0	100,00	99,94	0,06
233	3079	3079	0	0	100,00	100,00	0,00
234	2753	2753	0	0	100,00	100,00	0,00
All	109494	109241	163	255	99,76	99,85	0,39

The effect of all these complicated patterns is attenuated when considering the parameter of QRS localization h. One can see that using a unique threshold equals 0.1*max(h), great majority of QRS complexes are correctly localized.

Table 1 shows details of the detection of QRS complexes for each ECG signal. The analysis of the results of the table confirms that the algorithm can identify the position of the wave R with a reasonable precision. In fact, the sensitivity and the value of the positive predictivity P+ are calculated and give respectively 99.76% and 99.85%. The error rate varies between 0% and 2.31% with an average of 0.39%. More specifically, the algorithm proposed shows a better detection in terms of total rate error. In the light of all these findings, the fractional wavelet can be considered as an attractive solution for algorithm design of ECG signal treatment. Table 2 compares the performance of our algorithm with other well-known works.

Table 2. Comparison of QRS detector performance

Method	S_e (%)	P+ (%)	Er (%)
The proposed algorithm	99.76	99.85	0.39
J. Pan et al. [23]	99.75	99.54	0.71
S. Choi et al. [24]	99.66	99.80	0.54
Chen et al. [25]	99.47	99.54	0.98
Zidelmal et al [8].	99.64	99.82	0.54
Karimipour [26]	99.81	99.7	0.49

4. Conclusion

In this paper, an optimum QRS detector based on fractional wavelet is proposed. The choice of fractional order is guided by the fact that such filters allow some flexibility and accuracy thanks to a continuous adjustment of the wavelet parameters. Despite many sources of noise, the proposed detector shows great performance in QRS detection; and its efficiency is compared with some notorious works in the field. The sensitivity and the value of the positive predictivity are 99.76% and 99.85% respectively. The error rate is with an average of 0.39%. Work is currently in progress to implement this detector for real-time applications.

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