Selective Harmonic Eliminated Pulse Width Modulation (SHE-PWM)
Method using Genetic Algorithm in Single-Phase Multilevel Inverters

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Abstract: Numerous techniques have been used in the literature to produce switching signals in multilevel inverters (MLIs). Reducing the number of switching component in multilevel inverters is another area of interest in the literature. In this paper, a MLI with low switching number has been designed and worked successfully in harmonic elimination method. For this purpose, a novel genetic algorithm based SHE-PWM in a single phase 11-level inverter is presented in order to eliminate the switching signals. Also, total harmonic distortion (THD) analysis, which depends on switching angles for multilevel inverters, in a wide modulation range of 11-level inverter topology is presented. Hence, the switching angles are first estimated by Genetic Algorithm (GA) method, in which a certain number of harmonic components are eliminated. THD analysis is performed using both analytical and MATLAB simulation using calculated switching angles. The proposed method is applied to an 11-level inverter to eliminate 3th, 5th, 7th and 9th harmonic content in the range of modulation index 1.5 and 4. A simulink model which provides simulating and validating the accuracy, dynamics and continuity of the GA solution, was created. Experimental validation is not carried out in this study. Considering the simulation results and similar studies in the literature, the success of the reduced inverter topology and GA supported harmonic elimination technique was observed.

Keywords: Inverters; genetic algorithm; pwm; power conversion harmonics; harmonic distortion

1. Introduction

In recent years, multilevel inverters have been widely used due to its high efficiency, high voltage operation and low electromagnetic interference (EMI) in parallel with the developments in technology. In general, two basic strategies are considered in multilevel inverters. The first is the use of pulse width modulation (PWM) techniques to improve the output signal quality. Secondly, synthesizing of a large number of direct voltage (DC) sources to obtain high voltage. In the literature, there are various PWM techniques, such as traditional PWM, space vector PWM, sub-harmonic PWM, switching frequency optimum PWM, and selective harmonic elimination (SHE) PWM [1, 2].

In the SHE-PWM method, numerical techniques, which includes Newton Raphson, Groebner Bases and Symmetric Polynomial Theory, were used to solve nonlinear Fourier Transforms [3, 4]. This method finds the optimum point by derivative and ends at the local optimum point. It can only guarantee the result when selecting a suitable starting point. There are many multilevel inverter (MLI) topologies that are examined. Programmable PWM in multilevel inverters was applied to SHE method in [5]. Active harmonic elimination (AHE) method, which eliminates any number of high order harmonics of multilevel inverters using equal or unequal DC-voltage sources, was introduced in [6].

A new modulation technique, which combines the SHE method with Optimized Harmonic Stepped Waveform (OHSW) technique, is put forward in [7]. In [8], the SHE method using Walsh transformations was applied to a 7-level 3-phase classical multilevel inverter. Total harmonic distortion (THD) analysis was performed for MLIs of different levels in [9]. When the number of levels increase, it is stated that although the circuit complexity increases, the modulation index falls. In multilevel inverter, Memetic Algorithm (MA) is used to eliminate
harmonics [10]. In [11], a new single-phase cascaded MLI is proposed in order to reduce the number of switches.

SHE-PWM method developed with off-line meta-heuristic optimization algorithm is used in Four-legs DC / AC inverters [12]. In [13], Real-time solution for SHE is developed in seven-level inverter while Newton-Raphson and similar iterative methods are used as optimization methods. A high frequency multilevel transducer topology that provides electrical isolation at the input and output was proposed in [14]. In [15], to obtain switching angles in cascade MLIs generalized pattern search (GPS) optimization method is used. The technique that uses SHE and THD minimization approaches to eliminate low order harmonics, is presented in [16].

The study using the middle-level SHE pulse amplitude modulation (SHE-PAM) method is given in [17]. In this study, the relationship between the level increase in inverters and the number of eliminated harmonics is revealed. [18] proposes a new angle constraint SHE-PWM used in single phase inverter applications in order to eliminate all triple harmonics naturally. [19] explores SHE technique which is generally utilized in cascaded H-Bridge-inverter to succeed fundamental frequency switching. A study, which includes the reduction of the number of switching elements rather than control strategy in the cascade multilevel inverter where SHE method is used, is given in [20]. [21] presents the SHE technique, which uses general mathematical solutions for symmetric and asymmetric multilevel inverter topologies. In [22], eight different capacitor voltage balancing methods under SHE PWM conditions are compared and the differences are revealed.

In recent years, genetic algorithm (GA) has been designed in MLI for harmonic elimination. In AC-AC inverters, GA method was used to obtain PWM signals and compared with iterative methods [23]. The Generalized Hopfield Neural Network (GHNN) technique, another method for obtaining switching signals in AC-AC choppers, is presented [24]. Fuzzy Logic is used in [25] as artificial intelligence technique (ANN). A study using the GA and artificial neural network (ANN) technique is given in [26]. In [27, 28], SHE method using Gravitational Search Algorithm (GSA) technique is proposed. In [29], Particle Swarm Optimization (PSO) solves the harmonic elimination equations in MLIs.

In this article, the optimum switching angles in a single-phase 11-level inverter with five H-bridges are obtained using genetic algorithm technique which solves nonlinear equations faster than other techniques. The organisation of this article is as follows. In Section 2, the proposed MLI is introduced. In Section 3, harmonic elimination using the Genetic Algorithm method is described. Section 5 gives the simulation results, whereas a brief of summary of the conclusions are given in Section 6.

2. The Proposed Multilevel Inverter (MLI)

MLIs are exploited in a widespread manner by reason of their features such as reducing harmonic amplitudes, increasing output voltage and eliminating dependence on step-up transformers in industry. In recent studies, novel inverter topologies and modulation techniques are being studied. Diode-clamped, flying capacitor, and cascade H-bridge (CHB) inverter topologies are mostly used. The proposed MLI topology where the number of switches is substantially decreased compared to similar topologies is shown in Figure. 1a. The inverter level can be increased by using more switches indicated by T at the input and connected in series to the DC source.

The most common switching technique used in MLIs is step. The aim is to generate a low harmonic content signals which are similar to a sine wave shape. The Fourier series for the output voltage waveform illustrated in Figure. 1b is mathematically expressed by (1).

$$v(t)=a_0+\sum_{n=1}^{\infty} c_n \cos(nwt+\alpha_n)$$

(1)

where $\alpha_n$ is the initial phase for nth harmonic, $a_0$ is the value of dc component and $c_n$ is the value of nth harmonic components.
The produced voltage for different voltage of DC sources is given in (2).

\[
v(t) = \sum_{n=1,3,5,\ldots}^{\infty} \frac{4V_{dc}}{\pi n} \begin{bmatrix} v_1 \cos(n\alpha_1) \\ v_2 \cos(n\alpha_2) \\ \vdots \\ v_s \cos(n\alpha_s) \end{bmatrix} \sin(n\omega t)
\]

where \(s\) is the number of DC voltage sources connected per phase, \(v_1, v_2, \ldots, v_s\) and \(\alpha_1, \alpha_2, \ldots, \alpha_s\) are the voltage of DC sources and the switching angles, respectively. All switching angles must meet the condition given in (3).

\[
0 < \alpha_1 < \alpha_2 < \ldots < \alpha_s < \frac{\pi}{2}
\]

Five H-bridges per phase are required for 11-level inverter. In this case, five degrees of freedom \((s)\) is equal to 5. One angle degree is used to control the magnitude of the fundamental voltage, while the others eliminates harmonic components in the 5th, 7th, 11th, 13th and 15th order. If equal DC voltages case \((v_1=v_2=\ldots=v_s)\) is considered, the above statements can be rewritten as shown in (4).

\[
M = \cos(\alpha_1) + \cos(\alpha_2) + \cdots + \cos(\alpha_s)
\]

\[
0 = \cos(3\alpha_1) + \cos(3\alpha_2) + \cdots + \cos(3\alpha_s)
\]

\[
0 = \cos(5\alpha_1) + \cos(5\alpha_2) + \cdots + \cos(5\alpha_s)
\]

\[
0 = \cos(7\alpha_1) + \cos(7\alpha_2) + \cdots + \cos(7\alpha_s)
\]

\[
0 = \cos(9\alpha_1) + \cos(9\alpha_2) + \cdots + \cos(9\alpha_s)
\]
where M is the modulation index that described as the ratio between the fundamental output voltage and maximum obtainable voltage. Here, $\alpha_1, \alpha_2 \cdots \alpha_s$ are calculated by genetic algorithm (GA) which finds an optimum solution for (4).

3. Harmonic Elimination using GA

Genetic Algorithm (GA) method is used extensively in many different areas in the literature because of its ability to solve difficult problems easily. The most important feature of GA is that its foundations are based on genetic science and natural selection process. This method, inspired by biology, has been an important factor in the emergence of encouraging and efficient applications in many fields and especially in computer science.

\[ \alpha_1 \alpha_2 \alpha_5 \]

\[ 1011100101 \ 1000111100 \ 1100001100 \]

Figure 2. Structure of a five-angle chromosome

In the harmonic elimination method, it is necessary to find switching angles to provide the solution in the first stage. Here, each particular switching angle is expressed as a binary gene. A chromosome is formed by a combination of genes and in this study the chromosome size varies depending on the number of switching angles. For example, for five switching angles, there are five genes on a chromosome and each gene consists of a 10-bit binary number as shown in Figure 2.

Thus, each chromosome creates a solution for the problem. Population occurs when a certain number of chromosomes come together. For example, a population of 50 chromosomes initially offers us 50 possible solutions.

A new population is developed from this first population through analogue of certain basic genetic processes such as fitness, crossover, and mutation-based reproduction. In this study, all switching angles correspond to the population.

A number of chromosome selection is made during the reproductive phase with reference to natural selection. Subsequently, population members are selected according to the determined criteria and the suitability defined. The fitness score of candidate solution is evaluated by assigning a value to fitness function.

The flow diagram of the genetic algorithm method is given in Figure 3. In proposed GA method, the roulette wheel, inversion mutation and one-point crossover methods were used.

Roulette wheel is the simplest and the most frequently used selection approach. In this selection method, the whole roulette is partitioned into several sections. Each individual in the population are assigned a section of roulette wheel and has the probability to be selected in proportion to the fitness value. The fittest individuals have the largest section within the roulette wheel. The virtual roulette wheel is rotated and the individual corresponding to the section where the wheel stopped, is selected. This process is repeated to select the desired number of better individuals.

Inversion mutation method is used in permutation encoding. To carry out inversion, the two alleles from the chromosome selected for the mutation are randomly selected and the subsequences at these two alleles are inverted. Allele is the switching angle for one level.

In GA applications, the most commonly used crossover method is the one-point crossover technique. In this technique, single point fragmentation is used to produce offspring or child. It first selects two parents and then selects a random crossover point.
In the present study, it is desired to minimize the harmonics and the difference between the desired output voltage and the fundamental harmonic component. It is important to note that GA is working according to the maximization logic. For this reason, the fitness function must be determined accordingly. In this study, fitness function \( F \) is determined as follows.

\[
\begin{align*}
\text{hc} &= |c_3| + |c_5| + |c_7| + \cdots \\
\text{err} &= M_f - c_1 \\
F_a &= |\text{err}| + \text{hc} \\
F &= \frac{100}{F_a}
\end{align*}
\]

where \( h \) is the total magnitude of harmonic components. Equation (6) gives the error rate in the fundamental component \( (M_f) \). \( F_a \) is the total error.

The mating process is based on the idea of crossing the genes of two population members. The probability of having more of the most suitable chromosomes in the mating pool will be much higher. The selection process based on the use of a roulette wheel is quite complex. Here the percentage value on the roulette wheel varies according to the fit of the sequence. With this procedure applied to the first population, a new generation is produced. Here the chromosome with the highest fitness value will have a larger percentage in the wheel, while the opposite will be true for the unfit chromosome. Crossing describes a process in which a specified number of bits are exchanged between chromosomes. Mutation is another natural process in which any bit in each chromosome is replaced by its opposite. Crossing and mutation are performed according to 0.5 and 0.001 probability values, respectively. Eventually, the current population leaves its place to a new one, and this process continues until the termination criterion is reached.

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**Figure 3. Flowchart of proposed GA**
4. Simulation Results

In this study, MATLAB program is used to obtain the switching signals and apply them to the proposed inverter. The switching signals were obtained in the M-file environment using GA-assisted SHE-PWM technique. Table 1 lists the parameters used in the proposed GA technique. The 11-level inverter is drawn on the Simulink interface. Transformations have been made in order to use the switching signals obtained previously in M file interface in Simulink. R = 20 ohms and L = 10 mH are used as the equivalent load to the inverter output. RC snubber circuit is used to prevent peaks in the power element in the H Bridge structure. Powergui interface is used to measure TDH values of inverter output voltage.

Table 1. Proposed GA parameter

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population size</td>
<td>100</td>
</tr>
<tr>
<td>Coding</td>
<td>Binary</td>
</tr>
<tr>
<td>Structure of a chromosome</td>
<td>200 bit (for 5 angles)</td>
</tr>
<tr>
<td>Number of generations</td>
<td>500</td>
</tr>
<tr>
<td>Selection (reproduction) method</td>
<td>Roulette Wheel</td>
</tr>
<tr>
<td>Permutation encoding</td>
<td>Inversion mutation</td>
</tr>
<tr>
<td>Type of crossover</td>
<td>one-point crossover</td>
</tr>
<tr>
<td>Crossover rate (probability)</td>
<td>50%</td>
</tr>
<tr>
<td>Mutation rate (probability)</td>
<td>0.001</td>
</tr>
<tr>
<td>Termination criterion</td>
<td>500 iterations</td>
</tr>
</tbody>
</table>

As can be seen in Figure 4, the GA technique was achieved by modifying the signals, while the optimization process resulted in 500 iterations. Figure 4 shows that the intended GA method approaches the optimal result in a very short time. Therefore, the number of iterations can be reduced to less than 500.

![Figure 4. Obtaining switching signals with GA](image)

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As depicted in Figure 5, the switching angles (in radians) are plotted in the range of modulation index (M) 1.5 and 4. If M is 0, the output voltage reaches very small values and the inverter will not be used in this interval. If the modulation index is 5, the output voltage will be similar to the square wave. In the analyses, the GA method finds the switching moments in the region close to zero. Since the sine form at the output voltage is desired, results up to M = 4 are given. Figure 6 illustrates the output voltage and current waveforms for M = 4. When the current and voltage waveforms are considered, it can be seen that the output current coincides with the ideal form. If the number of levels is increased, it will look more like the ideal current shape. However, this leads to higher number of switches and increased costs in conventional MLI structures. In the proposed inverter topology, the number of additional switches will be very small compared to conventional structures.

Fast Fourier Transform (FFT) analysis of the output voltage for M = 4 is illustrated in Figure 7. Since the proposed MLI uses 5 switching angles, the basic component is kept constant while
eliminating the desired 4 harmonic components. The 3, 5, 7 and 9 harmonic components are eliminated as depicted in Figure 7.

Figure 7. Output voltage waveform and FFT for M=4.

Figure 8 shows variations of the output voltage THD versus the modulation range. THD value is high in low modulation index. Likewise, the THD value tends to increase slightly in the high modulation range. In the range of 3 and 4 of the modulation index, the harmonic distortion value is significantly reduced. If the number of inverter levels increases, more harmonic components can be eliminated as the number of independent variables will increase.

Figure 8. Output voltage THD vs modulation index (M) for the proposed MLI
Figure 9 illustrates THD variations of the output current in the modulation range. In the range of 3 and 4 of the modulation index, the harmonic distortion value is significantly reduced.

![Figure 9. Output current THD vs modulation index (M) for the proposed MLI](image)

The study on harmonic elimination with PWM method programmed in MLIs is given in [5]. Low harmonics component and THD graphs are given in this study. While low harmonic components are eliminated, the THD value has been reduced to 5% at high values of the modulation index. The results obtained from this study coincide with the results we have obtained. In another study conducted by the same researchers in [6], active elimination method was used and the THD value for large M values was presented as 5%. However, in terms of low harmonic components, the results obtained in this method were not as satisfactory as previous studies. A study using the SHE method based on the reference of Walsh transforms is given in [8]. Although the operation is 3-phase, an H bridge circuit is used for each DC level. For this reason, reducing the number of switches stands out as an advantage over most of the studies in the literature. In addition, in this study, low switching components were eliminated as in our study. Another study using H bridge for each inverter level is given in [9]. It is in parallel with the results obtained in our study with THD and harmonic components. The study using Memetic Algorithm as the SHE method is given in [10] and the THD value obtained is above the findings in our study. In other similar studies [11-14], the application aspect has been highlighted, but no detailed results have been given on eliminated harmonic components and THD. Finally, it was observed again that the results obtained in our study with the MLI study using the Staircase Modulation technique were overlapped.

As a result, the sensitivity of the simulation interfaces used today largely overlaps with the application results. For this reason, the simulation results we obtained in our study show great similarities with the studies performed. On the other hand, the use of low key number for MLI in the study makes the study come to the fore.

**5. Conclusion**

In this study, the SHE method assisted with GA in order to generate switching signals for MLIs fed from adjustable DC sources is investigated. Considering the nonlinearity of the harmonic elimination problem, the GA method, which provides a fast solution to be applied to a low switching number inverter circuit, is aimed in order to obtain the ladder output waveform of the inverter. The simulation results show that this research can be applied to the harmonic elimination problem in order to effectively eliminate the lowest harmonics and keep the fundamental component of the output voltage at the desired level. The proposed method’s validity has been demonstrated by simulations performed with MATLAB software. The results coincide with the
experimental studies in the literature. As a result, the designed system is recommended for high power applications that may be affected by low order harmonics and high THD.

6. References


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