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Abstract: In Software-Defined Networking (SDN), even though centralized information on network condition is available at the controller, this information is not used to improve network condition when congestion happens. SDN requires policy embedded in the controller to manage its network, e.g., strategy for network resource allocation. In this paper, we propose an optimal rate allocation schemes to support congestion control in SDN. Congestion control and rate allocation are like two sides of the same coin. Optimal rate allocation can reduce congestion probability, such that a complicated congestion control is not required. This rate allocation is based on mathematical optimization using three optimization criteria, i.e., minimization on mean transmission time, minimization on standard deviation, and allocation based on proportional rate allocation. The minimization problem for mean and standard deviation are solved using Lagrange method, while proportional rate allocation problem is solved using linear equation. The simulation provides better performance compared to rate allocation schemes without rate information.

Keywords: rate allocation, load distribution, congestion control, SDN.

1. Introduction

Congestion control is an important aspect on the telecommunication network optimization. Network congestion design focuses on how we should manage network resources. The purpose is to achieve good performance for user satisfaction. At present, there are various methods to manage network congestion. The methods are such as route setting [1, 2], admission control [3, 4], load sharing [5], data rate setting [6, 7, 8]. It is interesting to observe that congestion control and rate allocation are like two sides of the same coin. Rate allocation will correspond to better network congestion. However, managing network for better congestion leads to a problem of how to allocate rate correctly.

On the end-to-end congestion control, the rate allocation for each flow depends on the endhost congestion control mechanisms for all competing flows. If the sender transmits too fast, it will result in accumulating data in the network; on the other hand, if the sender transmits too slow, the network is underutilized. On this mechanism, the rate information between the sender and the receiver is often not known a priori; the competing flows are given the equal rate. If the congestion control mechanism can get rate information, then the transmission rate of the sender can be adjusted to the network conditions appropriately. Even if the routers could participate more actively in rate distribution, the network would be more robust and could accommodate more diverse users [9].

In traditional network frameworks, such as transfer control protocol (TCP)-based network [10], many researchers had investigated on how to optimize the transmission rate efficiently. TCP's model of allocation assumes that rate should be shared equally among contending flows. However, for an increasing number of networks such as data centers and private wide area networks (WANs), such an allocation is not a good fit [11]. The limitation of rate allocation on traditional network is usually based on short time feedback from the network to the users.

Consequently, it is difficult to design network with global optimal for long future operation of network as global network information is not known [12].

Another approach in network optimization is to use distributed approach [13]. The distributed approach data on network condition is used by various network components. This information is used for network design. The distributed approach, however, has limitation for network congestion management. These limitations, for example, include the usage of local information and reactive short time reaction. As a result of data collection, a distributed approach has a longtime response and thus less efficient.

Software-Defined Networking (SDN) is a network with a new paradigm, separating control logic from network infrastructure so SDN can change network policy through centralized programmable controller [14]-[15]. In the SDN, the controller translates the network management policies into packet forwarding rules and passes them to network devices, such as switches and routers. This mechanism reduces the complicated process in the data plane, and it facilitates network with better management. The critical areas of SDN bandwidth management are rate availability calculation and how to divide this rate based on the controller's policy.

Now, the problem of rate allocation has evolved in a variety of techniques both in the traditional network as well as the SDN architecture. In traditional networks, the problem of optimal rate allocation and congestion control for computer networks was spearheaded by F. Kelly [16]. Optimization was solved by Lagrange optimization theory. The optimization goal of each flow could determine its rate. The rate was based only on the "price" marked by the network. The main drawback of Kelly's scheme was all of the flows are priced equally. Then, several approaches had been proposed to overcome rate allocation problems. One of the methods was proportional rate sharing scheme [17]. This scheme was introduced by D. Luong et al. in 2001. Despite the findings above, there were no general systematic methods found easier to make the analysis work of rate sharing [18].

Meanwhile, several researchers have also conducted some studies for rate allocation and controlling bottlenecks in Software-Defined Network. Various approaches were offered to data center performance improvements such as the literature of [11], [19]-[21]. In [11], D. Bharadia presents NUMFabric, a method for rate allocation in the data center. The policies in NUMFabric were design based on the classic Network Utility Maximization (NUM) [21] framework which allows per-flow resource allocation preferences to be expressed using utility functions. In [19], Y. Lu and S. Zhu have designed the SDTCP, a software-defined network (SDN)-based TCP congestion control mechanism. This mechanism used a centralized method centralized control method and the global view of the network, to solve the TCP incast problems. Next, P. M. Mohan [20] proposed BASIS, a solution based on Bayesian inference for providing proportional Quality of Service (QoS) guarantees to tenants in a data center network. In BASIS, the rate of an outgoing congested link will be shared among the competing flows in proportion to the weights chosen by them. On another study proposed by J. M. Wang et al. in [21], they presented MCTEQ. They introduced a resource allocation approach with three classes that classes focus on the delay of high priority traffic.

The network resource allocation scheme was associated with the routing scheme on the SDN was proposed by M. M. Tajiki et al. [22]. They performed mathematical formulation for reallocation of QoS-aware resources. This formulation which it was allocated in a software-defined network (SDN) was based on traffic prediction. In [22] each network resource was adjusted for traffic prediction. The purpose was to reduce packet loss and increase throughput. A similar scheme for wireless SDN had been proposed by [23] and [24]. So far, most of the proposed worked for resource allocations in SDN were related to routing.

In this paper, we propose a rate allocation to support congestion control. This scheme is designed for the SDN framework. The central component of the proposed scheme is the model used to distribute the available rate of a predefined path on the path selection mechanism. A predefined path is a sequential link in a network connecting source to destination. The available rate is intended for the number of sources that the controller has allowed through that path. The main problem addressed in this research is how to allocate the source sending rate for faster

delivery. The network resource allocation problem is formulated mathematically with three optimization objectives, which are the minimization of sending time, minimization of standard deviation of sending time, and proportionally adding rate resource according to initial sending rate. This paper is arranged as follows. Section 2 discusses the related works on rate allocation in traditional networks and SDN. Section 3 presents the detail of the proposed method for optimal rate allocation and congestion controlled adaptive models. Simulation results for various scenarios are given in Section 4. Finally, Section 5 concludes this paper with summary of overall finding and the future outlook.

2. System Model

We consider the SDN as illustrated in Figure 1. We model the SDN data plane as a directed graph G(V, E), where V is the number of vertices (nodes), and E is the number of links. O is an ingress switch that receives incoming traffic from the network and transmits outgoing traffic to the network, where $O \subseteq V$. D is an egress switch, where $D \subseteq V$. A link from the $i \in V$ node to the $j \in V$ node is denoted as $(i, j) \in E(i \neq j)$. A set of Flow $F = \{f_i\}$ from source O is passed to the network toward destination D, has r_i sending rate. A path is defined as a sequence of nodes connecting O and D.



Figure 1. System Model.

In order to put the allocation problem on SDN, we first need to collect some network parameters quantitatively. In this paper, we assume that the network parameter to be distributed optimally is the available rate which is denoted as R_p . In order to calculate available rate R_p , consider a simple SDN network as shown in Figure 1. In this figure, the network status indicators are collected centrally by a controller ζ . The controller ζ shares this information to the ingress switch to manage the user traffic flows. By knowing the network information, the controller ζ could determine available paths for traffic flow in the network and calculate the available rate for each path simultaneously.

Let us consider a specific path P in the network that connects user traffic from source point to the destination point. Let this path consist of H links that connect to each neighboring switch or router in this path. The capacity of each link, denoted as C_i is defined as the maximum rate of the *i*-th link.

Sofia Naning Hertiana, et al.

Let us denote the link utilization of each link as

$$\rho_i = \frac{\lambda_i}{\mu_i} \tag{1}$$

where λ_i is the average incoming traffic rate, and μ_i is the average service rate at the node at that link.

Consider a network path consisting of a sequence of *H* links modeled as *H* successive queues. Assuming that the utilization of successive queues are uncorrelated, then the end-to-end utilization of the system, ρ , can be expressed as $\rho = 1 - \prod_{1 \le i \le H} (1 - \rho_i)$, [25] and the available rate on that path can be calculated as

$$R_{p} = \min_{i=1...H} C_{i} \cdot \left(1 - \left(1 - \prod_{1 \le i \le H} (1 - \rho_{i}) \right) \right)$$
(2)

The controller ζ can calculate this R_p and acknowledges it regularly to the ingress switch. The ingress switch, on receiving this information, can distribute this R_p to the connected users using a certain mechanism. The value of R_p has to be distributed according to a certain technique to the connected users. The distribution technique as proposed in this paper is described in the following section.

3. Proposed scheme



Figure 2. Rate allocation model.

The rate allocation is aimed at dividing the available rate of a path to the number of sources allowed by the controller to pass through the path. Each user gets the rate allocation is r_i . To calculate r_i we consider a simplified system model as illustrated in Figure 2. A path has rate path R_p . Ingress switch will be distributed and assigned to each user according to the optimal solution of rate allocation problem. In this paper, we propose three rate allocation schemes, namely minimization transmission average, minimization standard deviation and proportional allocation.

A. Optimization Formulation

Assume that there are N users with the initial traffic $v_1, v_2, ..., v_N$ that flows path P. Suppose that each source initially sends data at sending speed of $k_1, k_2, ..., k_N$ respectively. Without increasing or decreasing these speeds, then each source is expected to finish its data after time

$$\tau_i = \frac{v_i}{k_i} \tag{3}$$

From the network point of view, the mean (T_m) and standard deviation (S_m) of the transmission time of all users can be written as

$$T_m = \sum_{i=1}^N \tau_i = \sum_{i=1}^N \frac{v_i}{k_i}$$
(4)

and

$$S_m = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (T_m - \tau_i)^2}$$
(5)

respectively.

With the additional resources of R_p in the network, then this resource can be distributed to each source so that each source gets an additional rate. If the rate increment for each user is r_i , then the new the transmission rate \bar{k}_i for each source becomes

$$\bar{k}_i = k_i + r_i \tag{6}$$
and the sum of all rate increment r_i is R_p

 $\sum_{i=1}^{N} r_i = Rp. \tag{7}$

Using this additional rate r_i , the transmission time of each source becomes

$$\overline{\tau}_i = \frac{v_i}{\overline{k}_i} = \frac{v_i}{(k_i + r_i)} = \tau_i \frac{k_i}{k_i + r_i}$$
(8)

$$\bar{T}_m = \sum_{i=1}^N \bar{\tau}_i = \sum_{i=1}^N \tau_i \frac{k_i}{k_i + r_i}$$
(9)

and

$$\bar{S}_m = \sqrt{\frac{1}{N} \sum_{i=1}^N (\bar{T}_m - \bar{\tau}_i)^2} = \sqrt{\frac{1}{N} \sum_{i=1}^N \left[\left[\sum_{i=1}^N \tau_i \frac{k_i}{k_i + r_i} \right] - \tau_i \frac{k_i}{k_i + r_i} \right]^2}$$
(10)

If the rate increment r_i is positive $\bar{\tau}_i \leq \tau_i$ for each $i = 1 \dots N$, then it implies that $\bar{T}_m \leq T_m$. The problem of choosing additional allocations of r_i can thus be formulated as an attempt to minimize the average transmission time, i.e., fastest transmission average and minimize standard deviation, i.e., fairness of transmission time for each source, or the proportional rate increment, where each source get rate increment according to their traffic volume. The rate allocation problem as focused in this paper can be formulated as allocation of R_p into r_i that minimize the average transmission time (Formulation 1) and standard deviation of transmission time (Formulation 2) and proportional distribution (Formulation 3). Mathematically, these three formulations can be written as follows.

Formulation 1: Minimization transmission average.

Mathematical formulation of minimization of transmission time can be expressed $\min \frac{1}{N} \sum_{i=1}^{N} \tau_i \frac{k_i}{k_i + r_i}$ (11) subject to $r_1 + r_2 + \dots + r_N = R_n$

Formulation 2: Minimization standard deviation.

Mathematical formulation of minimization of standard deviation time can be expressed

$$\min_{v} \sqrt{\frac{1}{N} \sum_{i=i}^{2} \left(\left(\frac{1}{N} \sum_{i=i}^{N} \frac{v_i}{\kappa_i + r_i} \right) - \frac{v_i}{\kappa_i + r_i} \right)^2}$$

$$subject \ to \quad r_1 + r_2 + \dots + r_N = R_p$$
(12)

Formulation 3: Proportional allocation.

The third allocation scheme is how to distribute the available rate so that each user obtain additional rate which is proportional to their initial traffic volume. Proportional allocation is a method of dividing available rate of R_p to each source according to proportional allocation ratio parameter ω . Where ω is a parameter that sets the value of proportional rate based on the source volume. This value is the same for each source. Mathematical formulation of minimization of standard deviation time can be expressed

Sofia Naning Hertiana, et al.

Assign
$$r_i = \omega \cdot \upsilon_i$$
 subject to $\sum_{i=i}^N r_i = R_p$ (13)

B. Solution of Formulation 1

First, we solve (11) using the Lagrange method. Let us denote the objective function as F(r) as

$$F(r_1, r_2, \cdots, r_i) = \frac{1}{N} \sum_{i=i}^N \tau_i \frac{\kappa_i}{\kappa_i + r_i} = \frac{1}{N} \sum_{i=i}^N \frac{\mathcal{G}_i}{\kappa_i + r_i}$$
(14)

and the constraint function as

$$G(r_1, r_2, \cdots, r_i) = r_1 + r_2 + \dots + r_N - R_p = 0$$
(15)

The gradient of objective function and constraint function can be written respectively as

$$\nabla F(r_1, r_2, \dots, r_N) = -\frac{\upsilon_1}{2(\kappa_1 + r_1)^2} \vec{a}_{r1} - \frac{\upsilon_2}{2(\kappa_2 + r_2)^2} \vec{a}_{r2} - \dots - \frac{\upsilon_N}{2(\kappa_N + r_N)^2} \vec{a}_{rN}$$
(16)
$$\nabla G(r_1, r_2) = \vec{a}_{r1} + \vec{a}_{r2} + \dots + \vec{a}_{rN}$$
(17)

where a
$$\vec{a}_{ri}$$
 denotes a unit vector at *i* direction. Combining the gradient of objective and constraint function using Lagrange method to obtain Lagrange function $L(r,\Box) = \nabla F(r) - \lambda \cdot \nabla G(r)$ and set the value to zero, we obtain

$$\left(-\frac{\upsilon_1}{2(\kappa_1+r_1)^2}+\lambda\right)\vec{a}_{r1}+\left(-\frac{\upsilon_2}{2(\kappa_2+r_2)^2}+\lambda\right)\vec{a}_{r2}+\dots+\left(-\frac{\upsilon_N}{2(\kappa_N+r_N)^2}+\lambda\right)\vec{a}_{rN}=0$$
(18)

Setting each vector component in left hand side of (18) to zero, and rewrite it into a set of linear equation, we can simplify

$$\begin{bmatrix} 1 & -\sqrt{\frac{\nu_{I}}{\nu_{2}}} & 0 & \cdots & 0 \\ 1 & 0 & -\sqrt{\frac{\nu_{I}}{\nu_{3}}} & \cdots & 0 \\ 1 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & 1 & \cdots & I \end{bmatrix} \cdot \begin{bmatrix} r_{I} \\ r_{2} \\ \vdots \\ r_{N} \end{bmatrix} = \begin{bmatrix} -k_{I} + \sqrt{\frac{\nu_{I}}{\nu_{2}}} \cdot k_{2} & -k_{I} + \sqrt{\frac{\nu_{I}}{\nu_{3}}} \cdot k_{3} & \cdots & R_{p} \end{bmatrix}^{T}$$
(19)

Using matrix notation, we can write (19) as

$$PR_{sl} = c \tag{20}$$

Where

$$\boldsymbol{P} = \begin{bmatrix} 1 & -\sqrt{\frac{\nu_{I}}{\nu_{2}}} & 0 & \cdots & 0 \\ 1 & 0 & -\sqrt{\frac{\nu_{I}}{\nu_{3}}} & \cdots & 0 \\ 1 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & 1 & \cdots & 1 \end{bmatrix}$$

$$\boldsymbol{R}_{sI} = \begin{bmatrix} r_{I} & r_{2} & \cdots & r_{B} \end{bmatrix}^{T}$$
(21)

and

$$\mathbf{c} = \begin{bmatrix} -k_1 + \sqrt{\frac{\nu_1}{\nu_2}} \cdot k_2 & -k_1 + \sqrt{\frac{\nu_1}{\nu_3}} \cdot k_3 & \cdots & R_p \end{bmatrix}^T$$
Solution of (20) is

$$\mathbf{R}_{sl} = \mathbf{P}^{-l} \mathbf{c}$$
(24)

The algorithm of Formulation 1 is shown in Algorithm 1.

Algorithm 1 : Pseudo code of Formulation 1 (Minimization on transmission
average)
Input: <i>Rp</i> scalar, <i>K</i> set, <i>V</i> set
Output: \overline{K} set as new rate allocation
Initialisation :
1: Form matrix <i>P</i> as in (21)
2: Form matrix <i>C</i> as in (23)
Calculation :
3: Calc Rate Increment Set $R = P^{-1} \cdot C$
New Rate Assignment :
4: For $\forall k_i \in K$ do
5: $\overline{k_i} = k_i + r_i$
6: end for
7: return <i>K</i>

C. Solution of Formulation 2

In order to solve (12), we first simplify the objective function as

$$F(r_1, r_2, \cdots r_N) = \sqrt{\frac{1}{N} \sum_{i=i}^N \left(\left(\frac{1}{N} \sum_{j=i}^N \frac{v_j}{\kappa_j + r_j} \right) - \frac{v_i}{\kappa_i + r_i} \right)^2}$$
(25)

Minimum value of the square root function occurs at the similar point as the argument of that square root. Therefore, we define the objective function as

$$F_{2}(r_{1}, r_{2}, \dots, r_{N}) = \frac{1}{N} \sum_{i=i}^{N} \left(\left(\frac{1}{N} \sum_{j=i}^{N} \frac{v_{j}}{\kappa_{j} + r_{j}} \right) - \frac{v_{i}}{\kappa_{i} + r_{i}} \right)^{2}$$
(26)

The constraint function of minimization as given in (23) for two variables function is can be written as

$$G(r_1, r_2, \dots, r_N) = r_1 + r_2 + \dots + r_N - R_p = 0$$
⁽²⁷⁾

Combine gradient of objective function $F_2(r_1, r_2, ..., r_N)$ and constraint function $G(r_1, r_2, ..., r_N)$ using Lagrange method and equating each vector component to zero, we obtain a set of linear equation in matrix form which is

$$\begin{bmatrix} I & -\frac{\upsilon_I}{\upsilon_2} & 0 & \cdots & 0\\ I & 0 & -\frac{\upsilon_I}{\upsilon_3} & \cdots & 0\\ I & 0 & 0 & \cdots & 0\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ I & I & I & \cdots & I \end{bmatrix} \cdot \begin{bmatrix} r_I\\ r_2\\ \vdots\\ r_B \end{bmatrix} = \begin{bmatrix} -k_I + \frac{\upsilon_I}{\upsilon_2} \cdot k_2 & -k_I + \frac{\upsilon_I}{\upsilon_3} \cdot k_3 & \cdots & R_p \end{bmatrix}^T$$
(28)
short notation we write

In short notation, we write

$$QR_{s2} = d \tag{29}$$

where

$$\boldsymbol{\varrho} = \begin{bmatrix} 1 & -\frac{\upsilon_1}{\upsilon_2} & 0 & \cdots & 0 \\ 1 & 0 & -\frac{\upsilon_1}{\upsilon_3} & \cdots & 0 \\ 1 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & 1 & \cdots & 1 \end{bmatrix}$$
(30)

$$\boldsymbol{R}_{s2} = \begin{bmatrix} r_1 & r_2 & \cdots & r_B \end{bmatrix}^T \tag{31}$$

and

$$d = \left[-k_1 + \frac{\nu_1}{\nu_2} \cdot k_2 - k_1 + \frac{\nu_1}{\nu_3} \cdot k_3 \cdots R_p \right]^T$$
(32)

Solution of (29) is

$$\boldsymbol{R}_{s2} = \boldsymbol{Q}^{-1}\boldsymbol{d} \tag{33}$$

The algorithm of Formulation 2 is shown in Algorithm 2.

Algorithm 2 : Pseudo code of Formulation 2 (Minimization on standard
deviation)
Input: <i>Rp</i> scalar, <i>K</i> set, <i>V</i> set
Output: \overline{K} set as new rate allocation
Initialisation :
1: Form matrix Q as in (30)
2: Form matrix <i>D</i> as in (32)
Calculation :
3: Calc Rate Increment Set $R = Q^{-1} \cdot D$
New Rate Assignment :
4: For $\forall k_i \in K$ do
5: $\overline{k_i} = k_i + r_i$
6: end for
7: return <i>K</i>

D. Solution of Formulation 3.

Solving the third formulation is easier than the previous cases and can be solved directly using linear programming. In this formulation, each source is expected to have additional rate which is proportional to the initial traffic volume of each source, that is

$$r_i = \omega \cdot \upsilon_i \tag{34}$$

where ω is a proportional ratio. Equation (34) can be considered as objective function. The constraint function $G(r_1, r_2, \ldots, r_N)$ is similar to previous two cases ((15) and (27)). Using the matrix form, we can collect the objective and constraint function as

 $\begin{bmatrix} I & 0 & 0 & \cdots & -\nu_{I} \\ 0 & I & 0 & \cdots & -\nu_{2} \\ 0 & 0 & I & \cdots & -\nu_{3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ I & I & I & \cdots & 0 \end{bmatrix} \begin{bmatrix} r_{I} \\ r_{2} \\ r_{3} \\ \vdots \\ \omega \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ R_{p} \end{bmatrix}$ (35)

In compact notation, we can rewrite (35) as

$$\boldsymbol{U} \cdot \boldsymbol{R}_{\rho} = \boldsymbol{e} \tag{36}$$

where

$U = \begin{bmatrix} 1 & 0 & 0 & \cdots & -b_{1} \\ 0 & 1 & 0 & \cdots & -v_{2} \\ 0 & 0 & 1 & \cdots & -v_{3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & 1 & \cdots & 0 \end{bmatrix}$ $R_{\rho} = \begin{bmatrix} r_{1} & r_{2} & \cdots & \omega \end{bmatrix}$ $C = \begin{bmatrix} 0 & 0 & \cdots & R_{p} \end{bmatrix}^{T}$ (38) $C = \begin{bmatrix} 0 & 0 & \cdots & R_{p} \end{bmatrix}^{T}$ (39) The solution of (36) is $R_{\rho} = U^{-1} \cdot e$ (40)		
	$\begin{bmatrix} I & 0 & 0 & \cdots & -D_I \end{bmatrix}$	
$U = \begin{bmatrix} 0 & 0 & 1 & \cdots & -\upsilon_3 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & 1 & \cdots & 0 \end{bmatrix}$ $R_{\rho} = \begin{bmatrix} r_1 & r_2 & \cdots & \omega \end{bmatrix}$ $c = \begin{bmatrix} 0 & 0 & \cdots & R_p \end{bmatrix}^T$ (39) The solution of (36) is $R_{\rho} = U^{-1} \cdot e$ (40)	$\begin{bmatrix} 0 & 1 & 0 & \cdots & -v_2 \end{bmatrix}$	(37)
$\begin{bmatrix} \vdots & \vdots & \ddots & \vdots \\ I & I & I & \cdots & 0 \end{bmatrix}$ $\boldsymbol{R}_{\boldsymbol{\rho}} = \begin{bmatrix} r_{I} & r_{2} & \cdots & \omega \end{bmatrix}$ $\boldsymbol{c} = \begin{bmatrix} 0 & 0 & \cdots & R_{\boldsymbol{\rho}} \end{bmatrix}^{T}$ (39) The solution of (36) is $\boldsymbol{R}_{\boldsymbol{\rho}} = \boldsymbol{U}^{-1} \cdot \boldsymbol{e}$ (40)	$\boldsymbol{U} = \left \begin{array}{cccccccccccccccccccccccccccccccccccc$	
$\begin{bmatrix} I & I & I & \cdots & 0 \end{bmatrix}$ $\boldsymbol{R}_{\rho} = \begin{bmatrix} r_{I} & r_{2} & \cdots & \omega \end{bmatrix}$ $\boldsymbol{c} = \begin{bmatrix} 0 & 0 & \cdots & R_{p} \end{bmatrix}^{T}$ $The solution of (36) is$ $\boldsymbol{R}_{\rho} = \boldsymbol{U}^{-I} \cdot \boldsymbol{e}$ (40)		
$\boldsymbol{R}_{\boldsymbol{\rho}} = \begin{bmatrix} r_1 & r_2 & \cdots & \boldsymbol{\omega} \end{bmatrix} $ (38) $\boldsymbol{c} = \begin{bmatrix} 0 & 0 & \cdots & R_p \end{bmatrix}^T $ (39) The solution of (36) is $\boldsymbol{R}_{\boldsymbol{\rho}} = \boldsymbol{U}^{-1} \cdot \boldsymbol{e} $ (40)		
$\boldsymbol{c} = \begin{bmatrix} 0 & 0 & \cdots & R_p \end{bmatrix}^T $ The solution of (36) is $\boldsymbol{R}_{\boldsymbol{\rho}} = \boldsymbol{U}^{-1} \cdot \boldsymbol{e} $ (39) (40)	$\boldsymbol{R}_{\boldsymbol{\rho}} = \begin{bmatrix} r_1 & r_2 & \cdots & \omega \end{bmatrix}$	(38)
The solution of (36) is $\boldsymbol{R}_{\boldsymbol{\rho}} = \boldsymbol{U}^{-1} \cdot \boldsymbol{e}$ (40)	$\boldsymbol{c} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{R}_p \end{bmatrix}^T$	(39)
$\boldsymbol{R}_{\boldsymbol{\rho}} = \boldsymbol{U}^{-l} \cdot \boldsymbol{e} \tag{40}$	The solution of (36) is	
	$\boldsymbol{R}_{\boldsymbol{ ho}} = \boldsymbol{U}^{-1} \cdot \boldsymbol{e}$	(40)

The algorithm of Formulation 3 is shown in Algorithm 3.

Algor	ithm 3 : Pseudo code of Formulation 3 (Proportional allocation)
In	nput: Rp scalar, K set, V set
0	Dutput: \overline{K} set as new rate allocation
In	nitialisation :
1: Fe	form matrix Q as in (37)
2: F	form matrix D as in (39)
С	Calculation :
3: C	Calc Rate Increment $\operatorname{Set} R = U^{-1} \cdot E$
Ν	lew Rate Assignment :
4: F	$\operatorname{Cor} \forall k_i \in K \operatorname{do}$
5:	$\overline{k_i} = k_i + r_i$
6: ei	nd for
7: re	eturn <i>K</i>

E. Numerical example

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In this subsection, we use the result of three formulations to solve a simple problem. We consider a network with two sources with traffic volume v_1 =300 volume unit and v_2 =500 volume unit. Let both sources send data with initial transmission rate of k_1 =4 and k_2 =4 transmission rate unit. We assume that the controller informs available rate of R_p = 10 transmission rate unit to ingress switch. We will calculate r_1 and r_2 according to Formulation 1, 2, and 3.

For Formulation 1, by solving (24), we obtain the optimal solution that minimize average transmission time to be $R_s = [r_1 \ r_2] = [3.86 \ 6.14]$. With this additional rate, the first user has total transmission rate of 7.86 transmission rate unit, while second user has 10.14 transmission rate unit. Therefore, time to transmit for first and second user is 38.2 time unit and 49.3 time unit respectively. The average transmission time is 43.7 time unit and the standard deviation of 7.84. Figure 3 shows the contour of objective function of this simple case which is $(r) = \frac{1}{2} \left(\frac{300}{4+r_1} + \frac{500}{4+r_2} \right)$, and the constraint function $G(r) = r_1 + r_2 - 10 = 0$. The solution of Formulation 1 is the touching point between the F(r) and G(r) which is at value of $r_1 = 3.86$ $(r_2 = 6.14)$.



Figure 3. Contour of objective function F(r) and constraint function G(r) of Formulation 1 correspond to $v_1 = 300$, $v_2 = 500$, $k_1 = 4$ and $k_2 = 4$.

For Formulation 2, using the data, we solve (33) to obtain $R_s = [r_1 \quad r_2] = [2.75 \quad 7.25]$. With this additional rate, the first user has total transmission rate of 6.75 transmission rate unit and second user has 11.25 transmission rate unit. Therefore, time to transmit for first and second user similar which is 44.4 time unit. The average transmission time is 44.4 time unit and standard deviation 0.

Finally, using Formulation 3, we solve (40) to obtain the optimal solution that proportional distribution which is $R_{\rho} = \begin{bmatrix} r_1 & r_2 \end{bmatrix} = \begin{bmatrix} 3.75 & 6.25 \end{bmatrix}$, distribution factor $\omega = 0.0125$. With this additional rate, the first user has total transmission rate of 7.75 transmission rate unit, while second user has 10.25 transmission rate unit. The transmission time for first and second user is 38.7 times unit and 48.75 time unit. The average transmission time is 43.7 time unit and the standard deviation of 7.12.

The result of these formulations in this simple example is summarized in Table 1.

	Initial Value			Formulation 1			Formulation 2			Formulation3		
	ki	Vi	ti	ri	k _i '	ť,	ri	r _i '	t _i '	ri	k _i '	t _i '
Source 1	4	300	75	3.86	7.86	38.2	2.75	6.75	44.4	3.75	7.75	38.7
Source 2	4	500	125	6.14	10.14	49.3	7.25	10.25	44,4	6.25	10.25	48.75
Average			100			43.7			44.4			43.7
Standard deviation			35.4			7.84			0			7.12

Table 1. Comparison of rate distribution using three formulations for two sources.



Figure 4. Average and standard deviation of transmission time as function of distributed rate r_1 .

As we see, using three formulations, there are significant improvement on average and standard deviation of transmission time.

Figure 4 shows the curve of average transmission time and standard deviation of transmission time as function of amount of distributed rate r_1 .

We have also calculated for three sources, the result of these formulations for three source is summarized in Table 2. As in the calculations for two sources, there are significant improvement on average and standard deviation of transmission time.

	Initial Value			Formulation 1			Formulation 2			Formulation3		
	\mathbf{k}_{i}	Vi	ti	\mathbf{r}_{i}	k _i '	t _i '	ri	ki'	t _i '	ri	k _i '	t _i '
Source 1	5	100	20	0.66	5.66	17.67	-1.43	3.57	28.01	1.43	6.43	15.56
Source 2	5	200	40	3.00	8.00	25.00	2.14	7.14	28.01	2.85	7.85	25.48
Source 3	5	400	80	6.33	11.33	1.33	9.28	14.28	28.01	5.71	10.71	37.35
Average			140			54			84.03			78.38
Standard deviation			30.55			6.84			0			10.91

Table 2. Comparison of rate distribution using three formulations for three sources.

4. Computer Simulation

In this section, we verify the effectiveness of the proposed rate distribution method based on three formulations that had been explained previously. In this simulation we assume there are 100 users transmitting data with the traffic volume randomly generated within in the range of 10 to 100 traffic volume unit. These users transmit at three different initial transmission rates which are small variance initial transmission rate (1-1.5 transmission rate unit), medium variance (1-5 transmission rate unit), and large variance (1-10 transmission rate unit). For each initial transmission rates, we investigate the performance of three formulations in the case of relatively small, medium, large available rate which are 0.1, 1 and 10 rate unit respectively. The simulation is performed using Monte Carlo method with repetition of 1000 times in each simulation.

A. Simulation result for small variance of initial transmission rate.



Figure 5. Comparison of remaining traffic volume as a function of elapsed time in the case of small variance in initial transmitting rate (a) small available rate (b) medium available rate (c) large available rate.

As we have introduced before, in initial transmission rate, the initial transmission rate (for small variance) of the 100 users are generated randomly within the range of 1-1.5 transmission rate unit. The total traffic volume is the sum of traffic from these 100 users. This total traffic volume is used as vertical axis of the simulation curve. The horizontal axis is the time elapsed from beginning of simulation. The relatively small variance in transmission rate represents a non-aggressive user's application such as VoIP or other homogeneous applications.

There are three available rate simulated in this condition, which are the small, medium and large available rate. The simulation results are depicted in Figure 5(a), 5(b), and 5(c) respectively. In Figure 5(a), the Formulation 3 gives the best transmitting efficiency as this formulation has a smaller elapsed time to complete all traffic of the users. The performance of Formulation 1 is slightly worse that Formulation 3, as the available rate is so small to significantly improves the efficiency of the system. Formulation 2 which is trying to minimize the standard deviation of transmission time performs worst compared to Formulation 1 and 2 even though this case has better fairness to all users. Overall, in this case, transmission time improve from about 80 times unit in the case of no adjustment to about 55 times unit in the case of Formulation 2.

In the case of medium available rate (Figure 5(b)) and large available rate, we observe similar trends to the case of small available rate with the elapsed time improve even better as expected. We can see Figure 5(a), 5(b), and 5(c), Formulation 1, Formulation 2 and Formulation 3, all of which show faster completion times than schemes without knowledge of rate availability information (no adjustment).

B. Simulation result for medium variance of initial transmission rate.

The simulation set up in this scenario is similar to those in previous simulation. In this simulation, the initial transmitting rate is medium which mean that user traffics consist of several types of applications. The result for small, medium and large available rate is depicted in Figure 6(a), 6(b) and 6(c) respectively.



Figure 6. Comparison of remaining traffic volume as a function of elapsed time in the case of medium variance in initial transmitting rate (a) small available rate (b) medium available rate (c) large available rate.

In the case of small available rate (Figure 6(a)), Formulation 1 has the shortest elapsed time, which means that Formulation 1 has the best efficiency as compared to Formulation 2 and Formulation 3. This situation can be understood as Formulation 1 has a better capability to rearrange the transmitting rate of various initial rates. In this simulation we observed that completion time of the proposed formulation is about 3 times (Figure 6(a)), 6 times (Figure (6(c))) and 60 times (Figure (6(c)) faster than without no adjustment scenario for small, medium, and large available rate.

C. Simulation result for large variance of initial transmission rate.

In this simulation, we generate the initial transmission rate in the range of 1 to 10 transmission unit. This simulation represents a various type of user's applications. The simulation results for small, medium and large available rate are depicted in Figure 7(a), 7(b), and 7(c).



Figure 7. Comparison of remaining traffic volume as a function of elapsed time in the case of medium variance in initial transmitting rate (a) small available rate (b) medium available rate (c) large available rate.

In the case of small available rate, Formulation 1 has the shortest elapsed time as shown in Figure 7(a). In this large initial transmission rate, Formulation 1 has a better capability to rearrange the transmission rate so that minimum average transmission is achieved. Formulation 3 on the other hand, has difficulty to rearrange various users' transmission rates as it can only add the rate proportionally. Formulation 2, on the other hand, has also limited capability to re-arrange transmission rate as it always tries to force rate fairness. In the case of medium and large available rate, the three formulations perform about similar as they now have better freedom to re-arrange transmission rates. The simulation result indicated that the proposed formulation has about the similar advantage to the completion time as those in the previous simulation as compared to the no adjustment.

D. Simulation result for distribution of the rate increment.

Rate allocation and congestion control can be viewed as the coin with two different sides both serve to ensure efficient use of network resources. Rate allocation method, in theory, aims to provide that the allocation of rate for the sender should not be higher than the value that can be shared to the sender. Congestion control functions to ensure that the sender rate should not exceed the network's ability to process the traffic load. The method we proposed has fulfilled the above statement, as shown in Figure 8(a) and Figure 8(b). Figure 8(a) shows the rate distribution in two sources with allocated rate r_i and r_2 . We observe that when R_p equals 10 transmission rate unit, volume source 2 (v_2) fix at 300 volume unit and volume source 1 (v_1) incremental from 0 to 1000 volume unit, we get result that $r_1 + r_2 \le R_p$. Similarly, in Fig. 8 (b), we observe at v_1 equals 300 volume unit and v_2 equals 500 volume unit, R_p incremental from 5 to 55 transmission rate unit, we get $r_1 + r_2 \le R_p$.

Allocating r_1 and r_2 improves the network completion time as shown in Figure 5, Figure 6 and Figure 7. In the case of Formulation 2 and Formulation 3 we get similar situation as shown in Figure 9 and Figure 10. Here, the total value of $r_1 + r_2$ is less than or equal to R_p .

Rate allocation method controls the sending rate by maintaining an available rate variable R_p based on three formulations. Under these three formulations, sender's transmission rate can be adjusted to increase or to decrease. Positive rate adjustment means an increase of transmission rate, while negative value means a decrease of transmission rate. The rate allocation method also controls that the total of transmission rate not to exceed the available rate. As shown in Fig. 8 (a) and 9 (a) the source 1 is given a rate with a negative value, meaning that source 1 must decrease its rate so the total rate less than or equal to available rate.



Figure 8. Distribution of the rate increment of Formulation 1 as function of (a) increasing volume source 1 (v_1) (b). Increasing available rate (R_p).



Figure 9. Distribution of the rate increment of Formulation 2 as function of (a) increasing volume source $1(v_I)$ (b). Increasing available rate (R_p) .



Figure 10. Distribution of the rate increment of Formulation 3 as function of (a) increasing volume source $1(v_1)$ (b). Increasing available rate (R_p) .

5. Conclusion

In this paper, we have proposed three formulations for a better distribution of available rate of the network. This method is proposed particularly for SDN network which has a capability to centrally monitor the network thus the network parameters can be easily obtained and manipulated. The two formulations are formulated mathematically to minimize average and standard deviation of transmission time. The last formulation is derived such that the available rate is distributed proportionally. From the simulation, we observed that Formulation 1, Formulation 2 and Formulation 3, have faster completion times than schemes without knowledge of rate availability information (no adjustment). The proportional distribution (Formulation 3) has better performance in the case of small variance transmission rate which corresponds to a homogenous user application. Formulation for average transmission time minimization (Formulation 1) generally performs well in the case of small, medium, and large variance of user initial rate. In particular, it has good performance in the case of medium and large variance in users' initial transmission rate as it has better capability to re-arrange unbalanced users' transmission rate. Formulation that minimize the standard deviation of transmission time (Formulation 2) perform worst as compared to the other two as it has a strict requirement to force the fair transmission rates. As the future work, it is necessary to extend the optimization process such that the techniques are applicable for multipath network and heterogeneous user applications.

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